Three Dimensional Strain Field Morphing

Han-Bing Yan, Shi-Min Hu, Ralph R. Martin

Tsinghua University, China
Morphing

- What is Morphing?
- Why some morphing results are natural while others not?
  - The key is the deformation process should be smooth and uniform.
- Problem: What is Deformation? How to define it in mathematics?
Deformation Definition?

- Other morphing methods
  - Have not given deformation definition before.

- In mechanics, a concept for describing shape deformation has been used for hundreds of years.

- That is:
  - Strain
Strain

- **What is Strain?**
  - Strain is a tensor quantity that describes shape deformation extent at each point.
  - Strain is a microscopic quantity.
  - In 3D, strain has 6 independent components.
\[\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 + \left(\frac{\partial z'}{\partial x}\right)^2 - 1 \\
\left(\frac{\partial x'}{\partial y}\right)^2 + \left(\frac{\partial y'}{\partial y}\right)^2 + \left(\frac{\partial z'}{\partial y}\right)^2 - 1 \\
\left(\frac{\partial x'}{\partial z}\right)^2 + \left(\frac{\partial y'}{\partial z}\right)^2 + \left(\frac{\partial z'}{\partial z}\right)^2 - 1 \\
\frac{\partial x'}{\partial y} \frac{\partial y'}{\partial y} + \frac{\partial y'}{\partial z} \frac{\partial z'}{\partial z} + \frac{\partial z'}{\partial y} \frac{\partial y'}{\partial z} \\
\frac{\partial x'}{\partial z} \frac{\partial z'}{\partial z} + \frac{\partial z'}{\partial y} \frac{\partial y'}{\partial y} + \frac{\partial y'}{\partial x} \frac{\partial x'}{\partial z} \\
\frac{\partial x'}{\partial x} \frac{\partial x'}{\partial y} + \frac{\partial x'}{\partial y} \frac{\partial y'}{\partial y} + \frac{\partial y'}{\partial x} \frac{\partial x'}{\partial y}
\end{bmatrix}\]
Strain

$$\varepsilon_i \text{ gives the local infinitesimal scaling along the } i \text{ axis direction at point } p.$$
Strain

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} = \frac{1}{2}\begin{bmatrix}
\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 + \left(\frac{\partial z'}{\partial x}\right)^2 - 1 \\
\left(\frac{\partial x'}{\partial y}\right)^2 + \left(\frac{\partial y'}{\partial y}\right)^2 + \left(\frac{\partial z'}{\partial y}\right)^2 - 1 \\
\left(\frac{\partial x'}{\partial z}\right)^2 + \left(\frac{\partial y'}{\partial z}\right)^2 + \left(\frac{\partial z'}{\partial z}\right)^2 - 1 \\
\frac{\partial x'\partial x'}{\partial y\partial z} + \frac{\partial y'\partial y'}{\partial y\partial z} + \frac{\partial z'\partial z'}{\partial y\partial z} \\
\frac{\partial x'\partial x'}{\partial z\partial x} + \frac{\partial y'\partial y'}{\partial z\partial x} + \frac{\partial z'\partial z'}{\partial z\partial x} \\
\frac{\partial x'\partial x'}{\partial y\partial y} + \frac{\partial y'\partial y'}{\partial y\partial y} + \frac{\partial z'\partial z'}{\partial y\partial y}
\end{bmatrix}
\]

\(\gamma_{ij}\) represents the relative change in angle between lines initially in the \(i\) and \(j\) directions at point \(p\).
Deformation Analysis with Strain

Figure 1. Linear Interpolation

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Strain is a quantitative tool in analyzing shape deformation. If we want the morphing process is deformation uniform, the strain change should be monotone.
3D Strain Field Morphing

- **Input:**
  - Source and target meshes
  - Consistent 3D tetrahedron meshes.

- **Continuous**
  - Assume all shapes all continuous model; they are only be approximately represented by discrete tetrahedron meshes.
Two fields

- Position field
  - All points position in the continuous model construct a position field.

- Strain field
  - Field to describe shape deformation between two shapes, which can be calculated from two position field.
3D Strain Field Morphing

- Steps
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field
3D Strain Field Morphing

- Steps
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field
Strain field calculation

- Finite element theory is used in strain field calculation.
- Linear tetrahedron element is used.

\[ \sum x = \sum N_i x_i \]
\[ \sum y = \sum N_i y_i \]
\[ \sum z = \sum N_i z_i \]
3D Strain Field Morphing

- Steps
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field
Strain field interpolation

- For small deformation, linear strain field interpolation can be used.

\[ \varepsilon^t = (1 - t)\varepsilon^n \]

- For large deformation, modified strain field interpolation is used.
Strain field interpolation

- Modified strain field interpolation

\[
\begin{align*}
\varepsilon'_{x} &= \frac{1}{2} \left[ t \sqrt{1 + 2\varepsilon_{x}^{n}} + (1 - t) \right]^2 - 1 \\
\varepsilon'_{y} &= \frac{1}{2} \left[ t \sqrt{1 + 2\varepsilon_{y}^{n}} + (1 - t) \right]^2 - 1 \\
\varepsilon'_{z} &= \frac{1}{2} \left[ t \sqrt{1 + 2\varepsilon_{z}^{n}} + (1 - t) \right]^2 - 1 \\
\gamma'_{yz} &= (1 - t)\gamma_{yz}^{n} \\
\gamma'_{zx} &= (1 - t)\gamma_{zx}^{n} \\
\gamma'_{xy} &= (1 - t)\gamma_{xy}^{n}
\end{align*}
\]
3D Strain Field Morphing

- **Steps**
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field
**Calculation Position field**

- Find the position field which has nearest strain field to the interpolated strain field.

\[ W = \sum_{i=0}^{n-1} \int (\varepsilon^n_i - \varepsilon^t)^T (\varepsilon^n_i - \varepsilon^t) d\Omega \]

- **Solver**
  - Gradient optimization method.
  - Nonlinear equation solver (Newton-Raphson method).
Results
Elephant
Figure 1. Linear Interpolation

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Twist
Demo

Helix

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Demo

Animal Morphing
Question?
Thanks!