Note on Industrial Applications of Hu's Surface Extension Algorithm

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Abstract. An important surface modeling problem in CAD is to connect two disjoint B-spline patches with the second-order geometric continuity. In this paper we present a study to solve this problem based on the surface extension algorithm [Computer-Aided Design 2002; 34:415–419]. Nice properties of this extension algorithm are exploited in depth and thus make our solution very simple and efficient. Various practical examples are presented to demonstrate the usefulness and efficiency of our presented solution.

Keywords: Differential geometry, skinning, partial differential equations, splines.

1 Introduction

B-spline surfaces are widely used in most industrial CAD systems. Diverse practical algorithms have been proposed for various operations on B-spline surfaces, such as knot insertion and removal, degree elevation and reduction, etc. See [3] for an overview. One operation — extending a B-spline surface to a target curve — is recently proposed in [2].

The Hu's algorithm in [2] extends a given B-spline surface S to a target curve and represents the extended surface S' in B-spline form. A nice property of this algorithm is that the shape and the parameterization of the original surface S are preserved. In this paper, we present a note to show that by using the surface extension algorithm in a novel way, an important industrial problem presented below can be efficiently solved.

In industrial practice, usually large surfaces of complex physical object are designed by many small patches smoothly joined together. In many design activities (below we list two main cases), the smooth joins are frequently broken and small gaps appear among the surface patches:

- The designer may not be satisfied with one patch on the surface and will delete it to design a new patch with various constraints. However, the new patch may not fit the boundary of old patches very well;

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– When the model transfers between different CAD system, due to the different data precision, small gaps may occur among patches.

So an important engineering problem is that, given two surface patches with a small gap in between (usually the gap is about 1mm-5mm), join two patches with G^0, G^1 and G^2 continuity, respectively. In this paper we present a simple and efficient solution with the surface extension algorithm to solve the G^2-joint problem.

At first glance, this problem can be solved by using the general surface blending algorithms [5]. However, it does not work in practice because: (1) due to the small gap, the blending surface, behaving as the smooth transitional surface among geometric objects, is heavily wrinkled; (2) adding an additional blending surface for each small gap will increase the number of fragments in the modeling surface.

Our solution is efficient in the fashion that we extend one surface patch to smoothly join the other one, such that no additional blending surface is created. Another nice property is that the shape and the parameterization of the two original patches will not be changed.

This paper is organized as follows. The Hu's surface extension algorithm is briefly reviewed in Section 2. Section 3 presents our solution to the G^2-joint problem with the Hu's surface extension algorithm. Section 4 shows some examples in industrial applications. Conclusion is given in Section 5.

2 Hu's Surface Extension Algorithm

A pth-degree B-spline curve is defined by

$$C(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i, \quad 0 \le u \le 1$$

where the knot vector is $U = \{\underbrace{0, \cdots, 0}_{p+1}, u_{p+1}, \cdots, u_n, \underbrace{1, \cdots, 1}_{p+1}\}$. Using the algorithm on page 577 of Ref. [3], the vector U can be unclamped at either end. The same curve after unclamping can be defined by

$$C(u) = \sum_{i=0}^{n} N_{i,p}(u)\widetilde{\mathbf{P}}_i, \quad 0 \le u \le 1$$

over the knot vector $U = \{\underbrace{0, \cdots, 0}_{p+1}, u_{p+1}, \cdots, u_n, 1, \underbrace{s, \cdots, s}_{p}\}$, $s \ge 1$. To extend the curve C to a target point $\mathbf{R}$, the parameter s is determined by the arc-length estimation

$$s = 1 + \frac{\|\widetilde{\mathbf{P}}_n - \mathbf{R}\|}{\sum_{i=0}^{n-p} \|C(u_{i+p+1} - C(u_{i+p})\|}$$

and the extended curve is represented as

$$C_1(u) = \sum_{i=0}^{n+1} N_{i,p}(u)\widetilde{\mathbf{Q}}_i, \quad 0 \le u \le 1$$

over the knot vector $U = \{\underbrace{0, \cdots, 0}_{p+1}, \frac{u_{p+1}}{s}, \cdots, \frac{u_n}{s}, \frac{1}{s}, \underbrace{1, \cdots, 1}_{p+1}\}$, where

$$\widetilde{\mathbf{Q}}_i = \begin{cases} \widetilde{\mathbf{P}}_i, & i = 0, 1, \cdots, n; \\ \mathbf{R}, & i = n+1 \end{cases}$$

A degree $p \times q$ B-spline surface is defined by

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) \mathbf{P}_{i,j}, \quad 0 \le u \le 1, 0 \le v \le 1$$

with knot vectors

$$\begin{aligned} U &= \{\underbrace{0, \cdots, 0}_{p+1}, u_{p+1}, \cdots, u_n, \underbrace{1, \cdots, 1}_{p+1}\} \\ V &= \{\underbrace{0, \cdots, 0}_{q+1}, u_{q+1}, \cdots, u_m, \underbrace{1, \cdots, 1}_{q+1}\} \end{aligned}$$

Given a target curve $C(v)$, by knot insertion and degree elevation algorithms [3], $C(v)$ can have the same degree q as the knot vector V of the surface $S(u,v)$. Denote the control points of $C(v)$ by $\mathbf{Q}_i$, $i = 0, \cdots, m$. To extend $S(u,v)$ in the direction of u to $C(v)$, it suffices to extend the $m+1$ B-spline curves

$$C_j(u) = \sum_{i=0}^{n} N_{i,p}(u) \mathbf{P}_{i,j}, \quad j = 0, \cdots, m$$

to the target points $\mathbf{Q}_i$ with the same parameterization.

3 Surface Blending Based on Extension

Refer to Fig. 1. The goal is to smoothly connect two disjoint patches with the second-order geometric continuity (G^2 for short), and without creating any additional patch fragments. Our basic idea is simple as follows.

Let two patches A and B be connected between boundary curves C_1 and C_4. By knot insertion and degree elevation, these two curves can have the same knot vector. First, two intermediate curves

$$\begin{aligned} C_2(v) &= \tfrac{2}{3}C_1(v) + \tfrac{1}{3}C_4(v) \\ C_3(v) &= \tfrac{1}{3}C_1(v) + \tfrac{2}{3}C_4(v) \end{aligned} \tag{1}$$

are created. The positions of C_2 and C_3 will be further optimized by surface fairness criterion as stated in Section 3.3. Then the surface A is extended three times in turn: first to C_2, then to C_3 and finally to C_4.

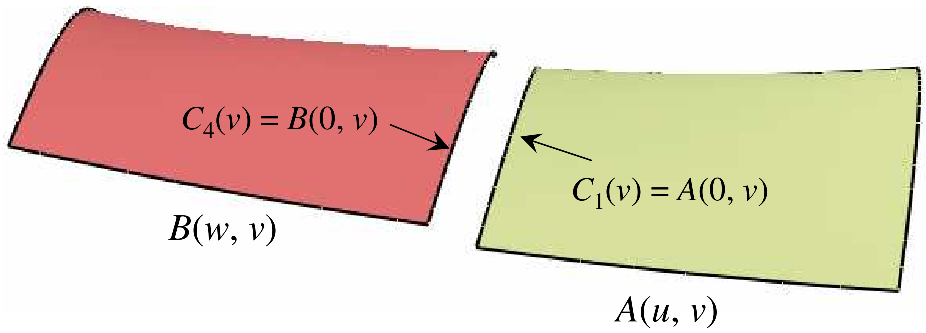


Fig. 1. Two disjoint patches that need to be G^2 smoothly joined along the boundaries C_1 and C_4

A nice property of Hu's surface extension algorithm is that the control points of C_2 and C_3 can be adjusted freely without changing the shape and parameterization of the original surface A. This property offers us sufficient degrees of freedom to make a G^2-join between A and B.

Let surfaces A and B be described by

$$A(u,v) = \sum_{i=0}^{m_1} \sum_{j=0}^{n} N_{i,p_1}(u) N_{j,q}(v) \mathbf{P}_{i,j},$$
$$B(w,v) = \sum_{i=0}^{m_2} \sum_{j=0}^{n} N_{i,p_2}(w) N_{j,q}(v) \mathbf{Q}_{i,j},$$

$$U = \{\underbrace{0,\cdots,0}_{p_1+1}, u_{p_1+1}, \cdots, u_{m_1}, \underbrace{1,\cdots,1}_{p_1+1}\}$$
$$V = \{\underbrace{0,\cdots,0}_{q+1}, v_{q+1}, \cdots, v_{n}, \underbrace{1,\cdots,1}_{q+1}\}$$
$$W = \{\underbrace{0,\cdots,0}_{p_2+1}, u_{p_2+1}, \cdots, u_{m_2}, \underbrace{1,\cdots,1}_{p_2+1}\}$$

Let the two surfaces be connected between boundary curves $C_1(v) = A(0,v)$ and $C_4(v) = B(0,v)$. The following is the algorithmic detail.

3.1 Surface Extension to a Target Curve

Refer to Fig. 1. By unclamping the knot vector U at the left end, the surface A can be extended left to a curve $C(v) = \sum_{j=0}^{n} N_{j,q}(v)\mathbf{R}_j$ with the knot vector V. The extended surface is represented as

$$\widetilde{A}(u,v) = \sum_{i=0}^{m_1+1} \sum_{j=0}^{n} N_{i,p_1}(u) N_{j,q}(v) \widetilde{\mathbf{P}}_{i,j} \tag{2}$$

where the new knot vector is

$$\widetilde{U} = \{\underbrace{-a,\cdots,-a}_{p_1+1}, \widetilde{u}_{p+1} = 0, \widetilde{u}_{p+2} = u_{p_1+1}, \cdots, u_{m_1}, \underbrace{1,\cdots,1}_{p_1+1}\} \tag{3}$$

$a > 0$ and is estimated by

$$a = \frac{1}{n+1} \sum_{j=0}^{n} \left(\frac{\|\mathbf{P}_{0,j} - \mathbf{R}_j\|}{\sum_{r=0}^{m_1-p_1} \|A_j(u_{p_1+r+1}) - A_j(u_{p_1+r})\|} \right)$$

$$A_j(u) = \sum_{i=0}^{m_1} N_{i,p_1}(u)\mathbf{P}_{i,j}, \quad 0 \le j \le n$$

By the surface extension algorithm [2] and the knot vector unclamping algorithm [3], the new control points in eq.(2) can be obtained by:

$$(i) \quad \widetilde{\mathbf{P}}^0_{i,j} = \mathbf{P}_{i,j}, \quad i = 0, 1, \cdots, p-1; \;\; j = 0, 1, \cdots, m_1$$

$$(ii) \quad \begin{cases} \widetilde{\mathbf{P}}^r_{i,j} = \widetilde{\mathbf{P}}^{r-1}_{i,j}, \quad i = r, \cdots, p-1 \\ \widetilde{\mathbf{P}}^r_{i,j} = \frac{\widetilde{\mathbf{P}}^{r-1}_{i,j} - \gamma \widetilde{\mathbf{P}}^r_{i+1,j}}{1-\alpha}, \quad i = r-1, r-2, \cdots, 0 \\ \text{where } \gamma = \frac{u_{p_1} - u_{p-r+i}}{u_{p+i+1} - u_{p-r+i}}, \quad r = 1, 2, \cdots, p-1, \quad j = 0, 1, \cdots, n \end{cases}$$

$$(iii) \quad \widetilde{\mathbf{P}}_{i,j} = \begin{cases} \mathbf{P}_{i-1,j}, \quad i = p+1, p+2, \cdots, m_1+1 \\ \widetilde{\mathbf{P}}^{p-1}_{i-1,j}, \quad i = 1, 2, \cdots, p \\ \mathbf{R}_j, \quad i = 0 \\ j = 0, 1, \cdots, n \end{cases}$$

The knot vector (3) can be rewritten as

$$\widetilde{U} = \{\underbrace{0, \cdots, 0}_{p_1+1}, \frac{a}{1+a}, \frac{u_{p_1+1}+a}{1+a}, \cdots, \frac{u_{m_1}+a}{1+a}, \underbrace{1, \cdots, 1}_{p_1+1}\}$$

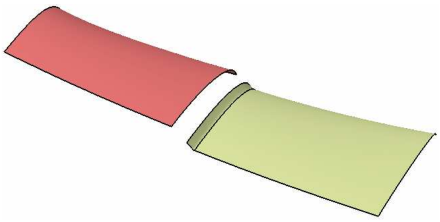
Refer to Fig. 2 (a-c). By extending the surface A from the boundary curve C_1 to the curves C_2, C_3 in eq.(1) and C_4 in turn, the two patches A and B are connected without changing the shape and parameterization of the original patches. To this end, the extended patch A is represented as

$$\widehat{A}(u,v) = \sum_{i=0}^{m_1+3} \sum_{j=0}^{n} N_{i,p_1}(u) N_{j,q}(v) \widehat{\mathbf{P}}_{i,j}$$

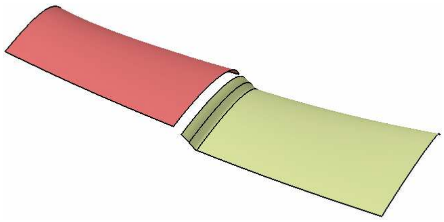
$$\widehat{U} = \{\underbrace{0, \cdots, 0}_{p_1+1}, \widehat{u}_{p_1}, \cdots, \widehat{u}_{m_1+3}, \underbrace{1, \cdots, 1}_{p_1+1}\}$$

3.2 Modify the Extended Surface to Meet G^2 Continuity

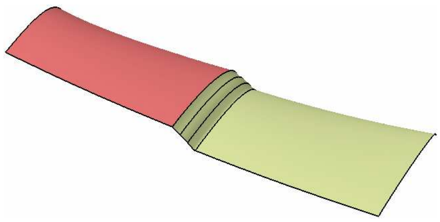
Refer to Fig. 2(c-d). By surface extension, we can adjust the second and the third rows (i.e., $\widehat{\mathbf{P}}_{1,j}$ and $\widehat{\mathbf{P}}_{2,j}$) of control points of the extended surface A to make a G^2 connection with surface B, while the shape of the original patches A and B does not change.



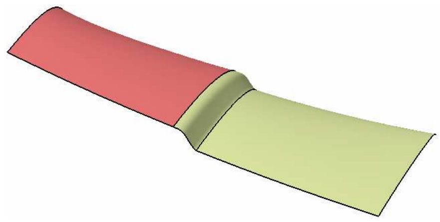
(a) Surface extension to C_2



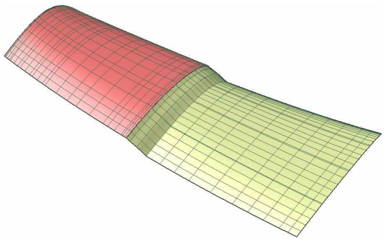
(b) Surface extension to C_3



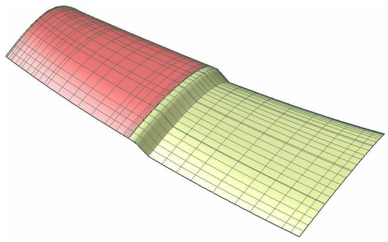
(c) Surface extension to C_4



(d) G^2-join surface A and B



(e) The net of control points of (c)



(f) The net of control points of (d)

Fig. 2. G^2-connect two disjoint patches in four steps

To connect surfaces A and B with G^2 continuity, the control points $\widehat{\mathbf{P}}_{1,j}$ and $\widehat{\mathbf{P}}_{2,j}$, $j = 0, 1, \cdots, n$ of A should be adjusted such that the following equation is satisfied:

$$\begin{cases} \widehat{A}_u(0,v) = \alpha B_w(0,v) \\ \widehat{A}_{uu}(0,v) = \alpha^2 B_{ww}(0,v) + \beta B_w(0,v) \end{cases}$$

where α, β are some constants.

For easy programming, we set $\beta = 0$ and the new positions of control points $\widehat{\mathbf{P}}_{1,j}$ and $\widehat{\mathbf{P}}_{2,j}$ of A are determined by

$$\begin{cases} \widehat{\mathbf{P}}_{0,j} = \mathbf{Q}_{0,j} \\ N'_{0,p_1}(0)\widehat{\mathbf{P}}_{0,j} + N'_{1,p_1}(0)\widehat{\mathbf{P}}_{1,j} = \alpha\left(N'_{0,p_2}(0)\mathbf{Q}_{0,j} + N'_{1,p_2}(0)\mathbf{Q}_{1,j}\right) \\ N''_{0,p_1}(0)\widehat{\mathbf{P}}_{0,j} + N''_{1,p_1}(0)\widehat{\mathbf{P}}_{1,j} + N''_{2,p_1}(0)\widehat{\mathbf{P}}_{2,j} = \\ \alpha^2\left(N''_{0,p_2}(0)\mathbf{Q}_{0,j} + N''_{1,p_2}(0)\mathbf{Q}_{1,j} + N''_{2,p_2}(0)\mathbf{Q}_{2,j}\right) \end{cases} \tag{4}$$
$$j = 0, 1, \cdots, n$$

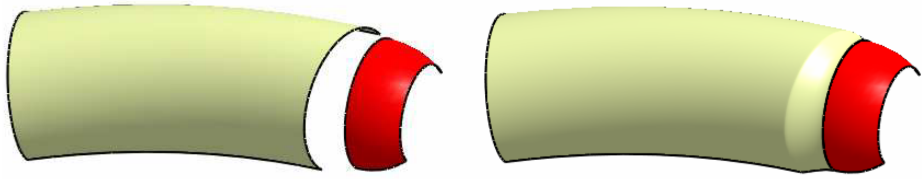


Fig. 3. G^2-connect two disjoint patches and minimize the surface fairness energy

The constant α offers us one more degree of freedom such that we can optimize the blending by minimizing the surface fairness energy.

3.3 Using Fairness Criterion to Optimize α

We use the standard second order energy

$$E = \iint A_{uu}^2 + 2A_{uv}^2 + A_{vv}^2 dudv \tag{5}$$

to measure the fairness of surface A [6]. By solving eq.(4), we have

$$\begin{aligned}
\widehat{\mathbf{P}}_{1,j} &= \tfrac{1}{N'_{1,p_1}(0)} \left[\alpha \left(N'_{0,p_2}(0)\mathbf{Q}_{0,j} + N'_{1,p_2}(0)\mathbf{Q}_{1,j}\right) - N'_{0,p_1}(0)\widehat{\mathbf{P}}_{0,j}\right] \\
&= \alpha M_{1,j} + N_{1,j} \\
\widehat{\mathbf{P}}_{2,j} &= \tfrac{1}{N''_{2,p_1}(0)} \left[\alpha^2 \left(N''_{0,p_2}(0)\mathbf{Q}_{0,j} + N''_{1,p_2}(0)\mathbf{Q}_{1,j} + N''_{2,p_2}(0)\mathbf{Q}_{2,j}\right)\right. \\
&\quad \left. -N''_{0,p_1}(0)\widehat{\mathbf{P}}_{0,j} - N''_{1,p_1}(0)\widehat{\mathbf{P}}_{1,j}\right] \\
&= \alpha^2 C_{2,j} + \alpha D_{2,j} + E_{2,j}
\end{aligned}$$

where $M_{1,j}, N_{1,j}, C_{2,j}, D_{2,j}, E_{2,j}$ are some constants. It can be calculated that energy $E(\alpha)$ in eq.(5) is a polynomial of degree four. Minimizing energy $E(\alpha)$ leads to solving a cubic equation, whose close-form formula can be found analytically [7].

4 Examples

By using formula (4) and solving eq.(5), our solution is very simple. The major advantage is that by extending one patch to G^2-connect the other patch, no additional patch fragment is created inbetween to blend the two disjoint patches; this nice property makes our solution particularly suitable for sewing two patches with small gaps. Two examples are shown in Figs. 2 and 3. An extreme case is illustrated in Fig. 4 in which two plane patches are G^2-connected.

In our solution, the boundaries of two patches to be connected do not need to be open. In Fig. 5, two tubes with closed profiles, generated by swept surfaces,

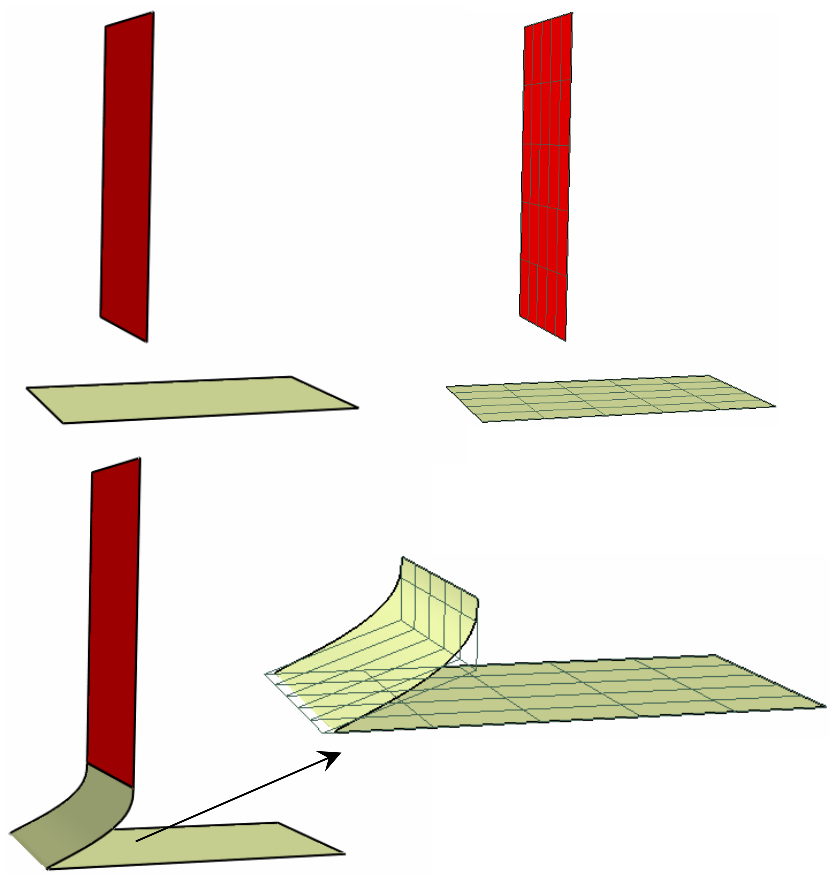


Fig. 4. G^2-connect two disjoint plane patches. The yellow patch is extended to meet the red plane patch; the right columns show the net of control points. By solving eq.(6), the optimal α is 7.63 to minimize the surface fairness energy.

are connected with G^2 continuity. To better illustrate the effect of surface fairness control by the simple parameter α, in these figures we show the faired surfaces with two different α values. Both tools of highlight lines [1] and isophotes [4] are used to inspect the surface quality (ref. Figs.5-6). The results clearly show the quality improvement in terms of surface fairness by minimizing energy (5) with one simple parameter α. Our method is simple and efficient because the exact minimization can be easily found by solving eq.(6).

An application of our solution to the surfaces' G^2-connection problem is shown in Fig. 6, in which a scoop model is generated by G^2-connecting the handle part and the container part. Both tools of highlight lines and isophotes are shown to demonstrate the fairness and smoothness of the generated scoop model.

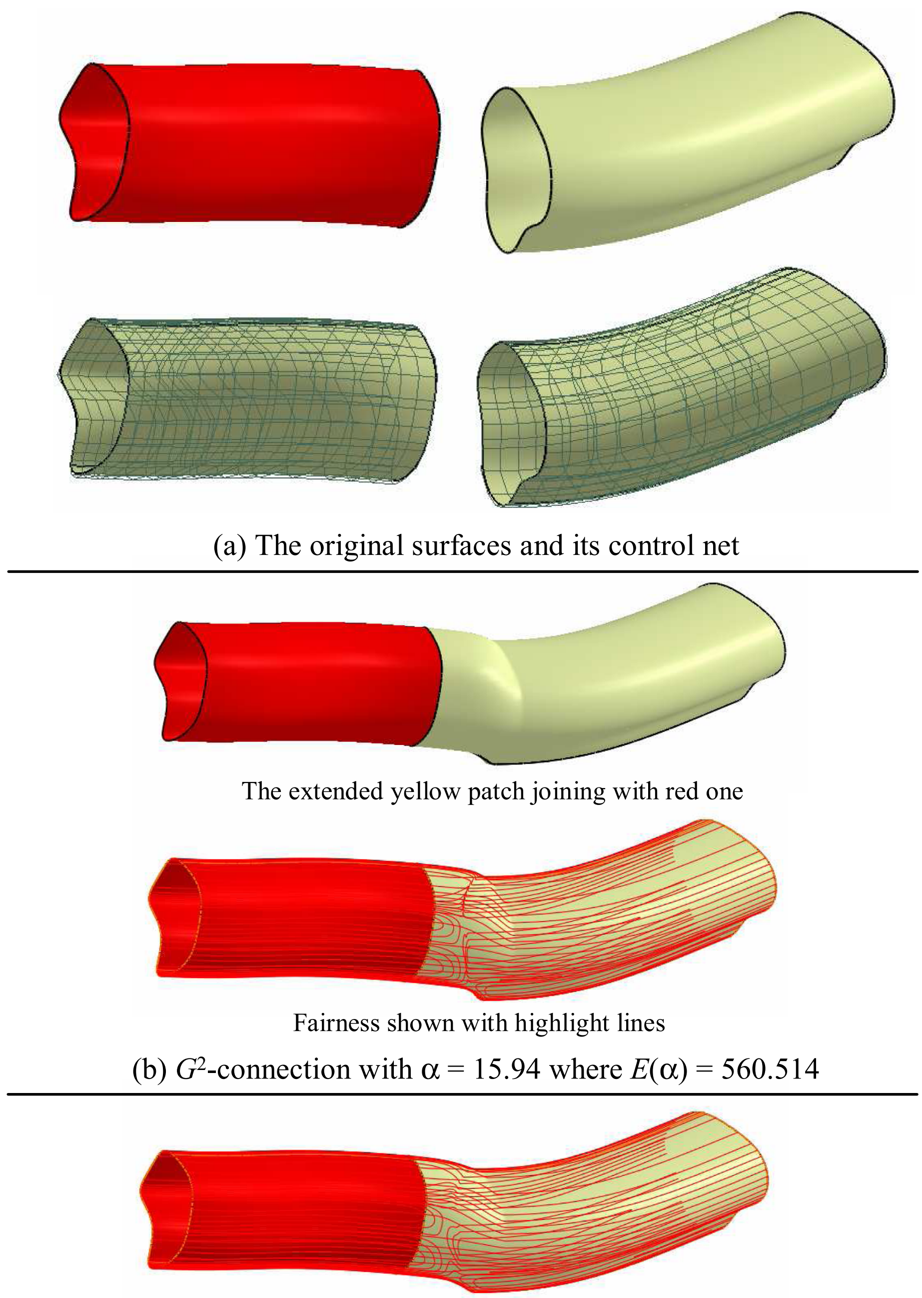


Fig. 5. G^2-connect two disjoint tubes. Surface fairness of different α values are illustrated and, highlight lines are used to inspect the surface quality.

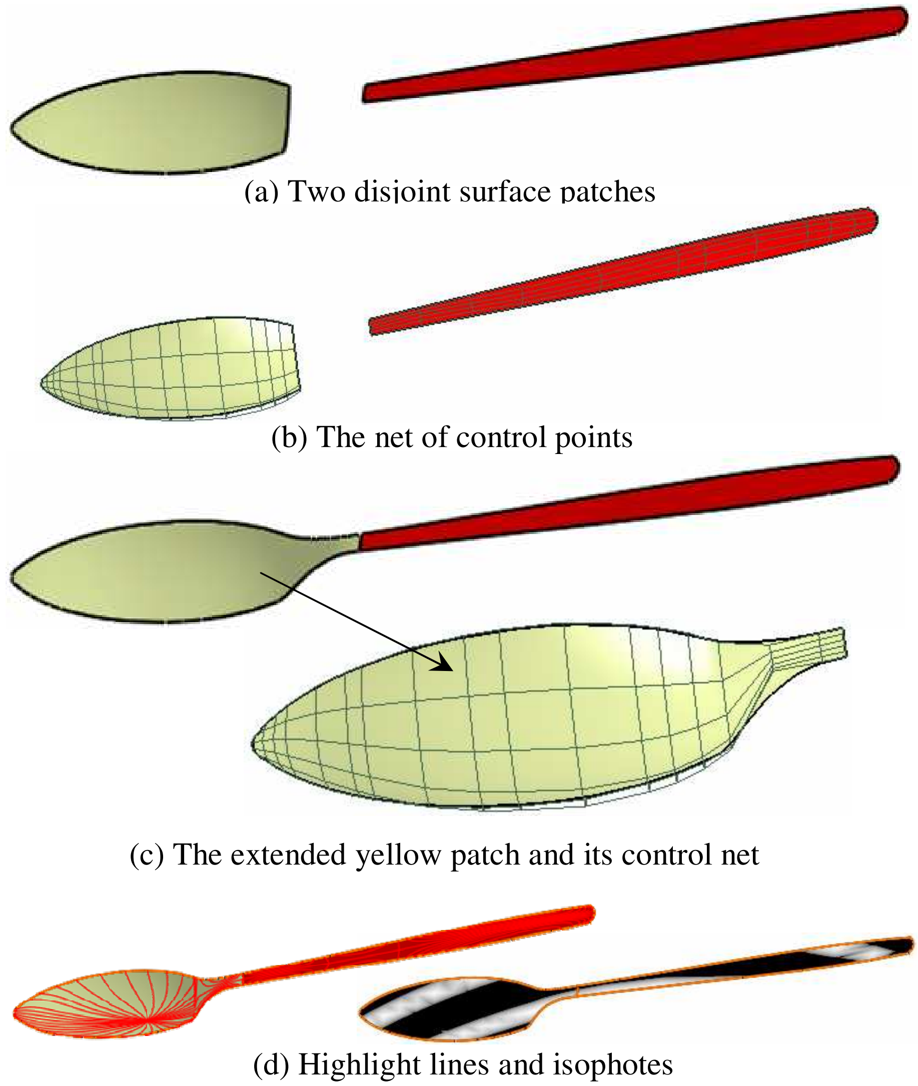


Fig. 6. Scoop modeling by surface extension with G^2 continuity

5 Conclusions

In this paper, we propose a simple and efficient method to connect two disjoint surface patches with G^2 continuity. Two major advantages are:

- the shape and parameterization of two original patches do not changes;
- no additional fragments are created between two disjoint patches; instead, the user can select one patch to extend and G^2-connect the other one.

We have successfully incorporated this method into a plugin module for a commercial CAD software. This module has been intensively tested and satisfied by industrial users.

Acknowledgments

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References

1. Beier, K.P., Chen, Y.: Hightlight-line algorithm for realtime surface-quality assessment. Computer-Aided Design 26(4), 268–277 (1994)
2. Hu, S.M., Tai, C.L., Zhang, S.: An extension algorithm for B-splines by curve unclamping. Computer-Aided Design 34, 415–419 (2002)
3. Piegl, L., Tiller, W.: The NURBS Book, 2nd edn. Springer, Heidelberg (1997)
4. Poeschl, T.: Detecting surface irregularities using isophotes. Computer Aided Geometric Design 1(2), 163–168 (1984)
5. Vida, J., Martin, R.R., Varady, T.: A survey of blending methods that use parametric surfaces. Computer-Aided Design 26(5), 341–365 (1994)
6. Wallner, J.: Note on curve and surface energies. Computer Aided Geometric Design (in press, 2007), doi:10.1016/j.cagd.2007.05.007
7. `http://mathworld.wolfram.com/CubicEquation.html`