

Supplemental Material

Some properties of exact geodesics on triangular surfaces

A real case is presented in the Example 4 in the report to draw the following observation:

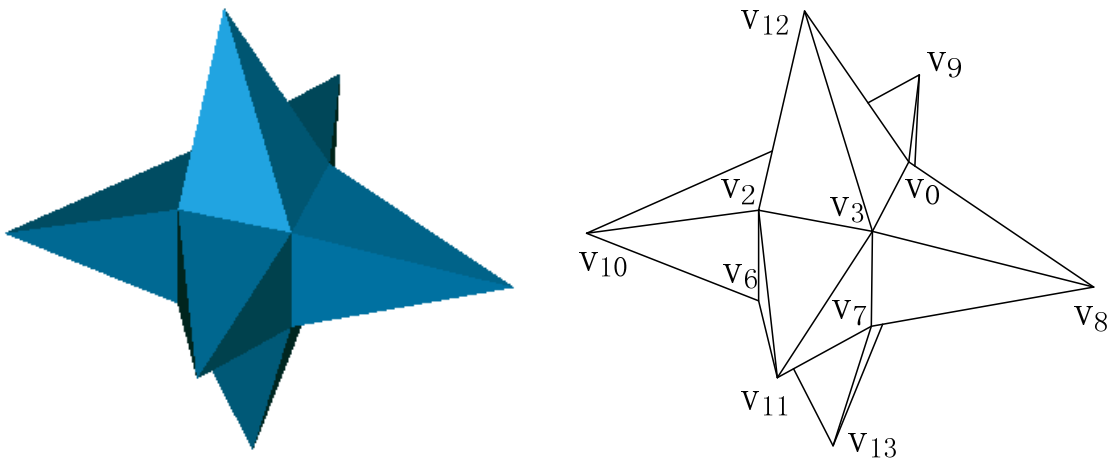
Observation 1. In the 1D edge subdivision structure in exact geodesic computation, the number of wedges on different edges in the triangle model varied severely and the distribution of wedges on each edge is highly non-uniform.

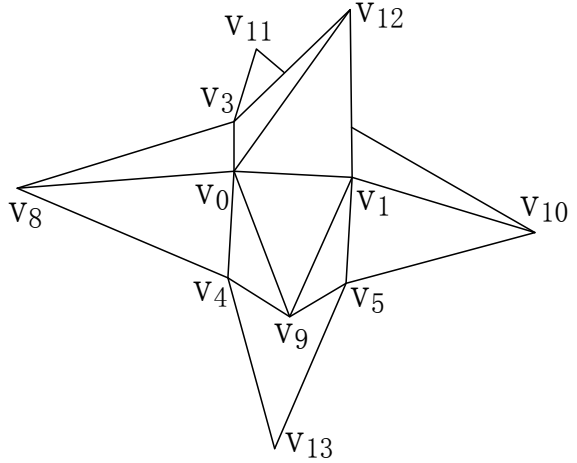
Observation 1 also forms the base of Observations 2 and 3 presented in the Section 6 in the paper:

Observation 2. Compared to non-uniform models, uniform models have statistically less wedges on the edges, whose distribution of wedges on each edge is more concentrated near the mean, and the running time is generally smaller.

Observation 3. If floating-point arithmetic is used, the average number of wedges per unit edge length is scale-dependent, i.e., it is inversely proportional to the average edge length of the model.

The completed Example 4 using the following triangle model is presented below:





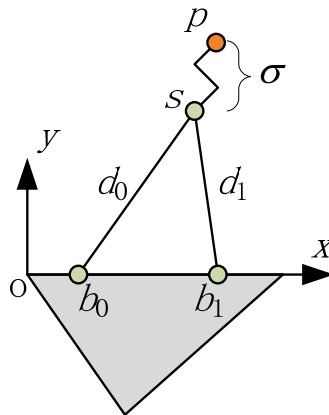
The vertex positions are:

$$v_0 = (0,0,0), v_1 = (0,1,0), v_2 = (1,1,0), v_3 = (1,0,0), v_4 = (0,0,1), v_5 = (0,1,1), v_6 = (1,1,1), v_7 = (1,0,1), v_8 = (0.5, -1.8, 0.5), v_9 = (-1.8, 0.5, 0.5), v_{10} = (0.8, 2.6, 0.8), v_{11} = (2.9, 0.2, 0.2), v_{12} = (0.1, 0.9, -1.5), v_{13} = (0.5, 0.5, 2.8)$$

The triangle indices are:

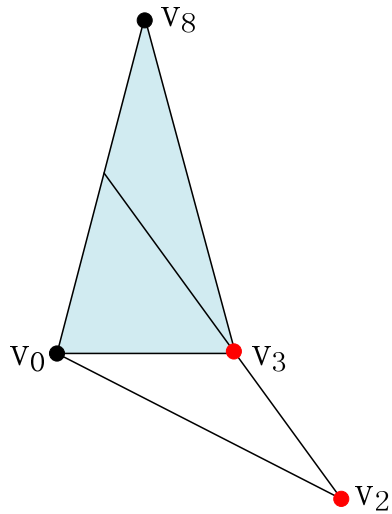
$$f_0 = (v_0, v_3, v_8), f_1 = (v_3, v_7, v_8), f_2 = (v_7, v_4, v_8), f_3 = (v_4, v_0, v_8), f_4 = (v_1, v_0, v_9), f_5 = (v_5, v_1, v_9), f_6 = (v_4, v_5, v_9), f_7 = (v_0, v_4, v_9), f_8 = (v_2, v_1, v_{10}), f_9 = (v_6, v_2, v_{10}), f_{10} = (v_5, v_6, v_{10}), f_{11} = (v_1, v_5, v_{10}), f_{12} = (v_2, v_6, v_{11}), f_{13} = (v_6, v_7, v_{11}), f_{14} = (v_7, v_3, v_{11}), f_{15} = (v_3, v_2, v_{11}), f_{16} = (v_0, v_1, v_{12}), f_{17} = (v_1, v_2, v_{12}), f_{18} = (v_2, v_3, v_{12}), f_{19} = (v_3, v_0, v_{12}), f_{20} = (v_5, v_4, v_{13}), f_{21} = (v_6, v_5, v_{13}), f_{22} = (v_7, v_6, v_{13}), f_{23} = (v_4, v_7, v_{13}).$$

The exact geodesic algorithm is run on this model with the source point locating at $v_2 = (1,1,0)$. The resulting subdivision on all the edges is shown blow, in terms of each face. The wedges in each edge are sorted and represented by the 7-tuple $(b_0, b_1, d_0, d_1, \tau, s, \sigma)$ with the meaning shown below:



where σ is the geodesic distance from pseudo-vertex s to the source $p = v_2 = (1,1,0)$, τ is true (false) if the visibility wedge comes from the outside (inside) of the face. If the wedge is degenerated into a line and coincides with the edge, τ is false.

Face 0: $f_0 = (v_0, v_3, v_8)$



There is one wedge at edge (v_0, v_3) :

$(0.0, 1.0, 1.778, 1.0, \text{true}, v_2, 0.0)$

There is one wedge at edge (v_3, v_8) :

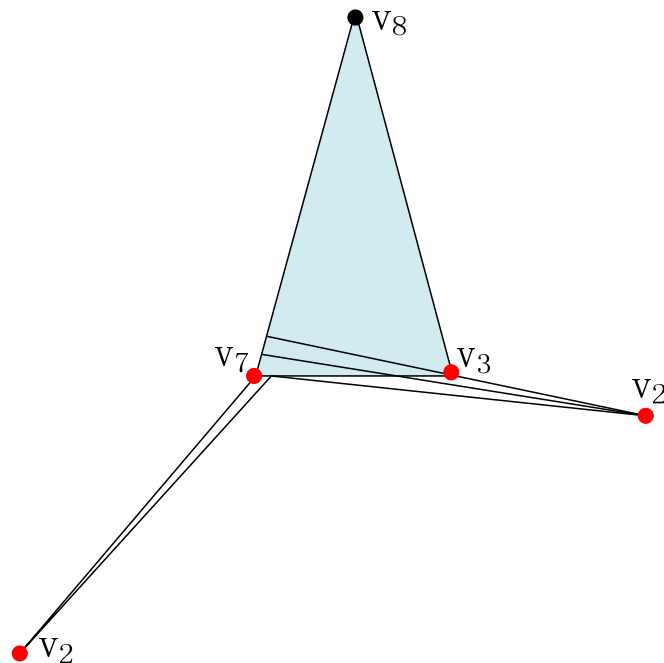
$(0.0, 1.934, 0.0, 1.934, \text{false}, v_3, 1.0)$

There are two wedges at edge (v_8, v_0) :

$(0.0, 0.88, 1.934, 1.251, \text{false}, v_3, 1.0)$

$(0.88, 1.934, 2.251, 1.778, \text{false}, v_2, 0.0)$

Face 1: $f_1 = (v_3, v_7, v_8)$

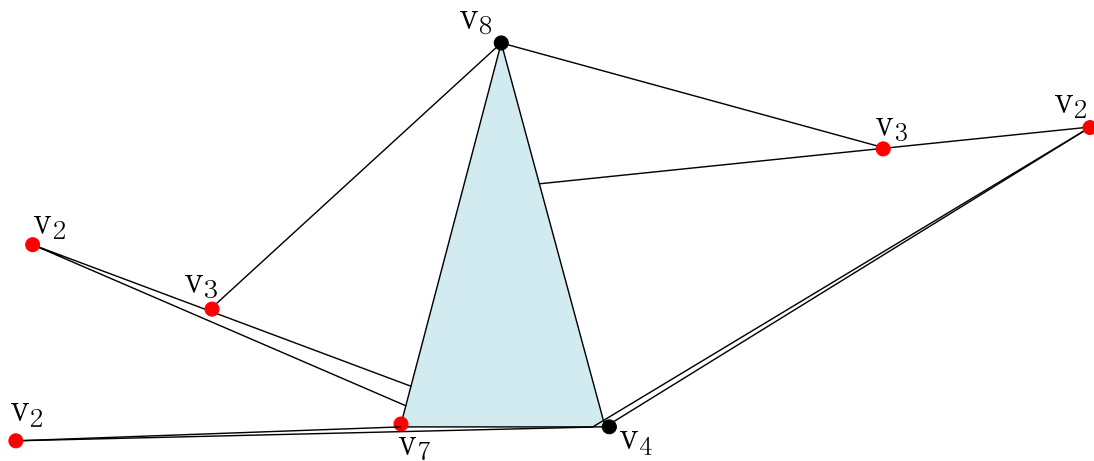


There are two wedges at edge (v_7, v_3) :
 $(0.0, 0.075, 1.865, 1.914, true, v_2, 0.0)$
 $(0.075, 1.0, 1.914, 1.0, true, v_2, 0.0)$

There are one wedge at edge (v_3, v_8) :
 $(0.0, 1.934, 0.0, 1.934, false, v_3, 1.0)$

There are three wedges at edge (v_8, v_7) :
 $(0.0, 1.727, 1.934, 0.967, false, v_3, 1.0)$
 $(1.727, 1.824, 1.967, 1.975, false, v_2, 0.0)$
 $(1.824, 1.934, 0.110, 0.0, false, v_7, 1.865)$

Face 2: $f_2 = (v_7, v_4, v_8)$

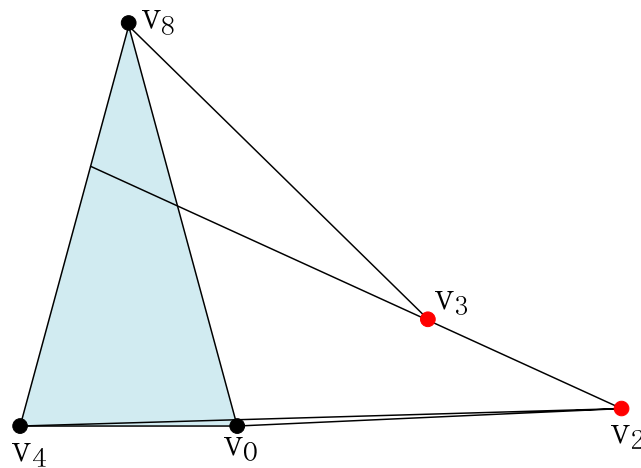


There are two wedges at edge (v_7, v_4) :
 $(0.0, 0.957, 1.865, 2.817, true, v_2, 0.0)$
 $(0.957, 1.0, 2.817, 2.778, false, v_2, 0.0)$

There are two wedges at edge (v_4, v_8) :
 $(0.0, 1.245, 2.778, 2.697, true, v_2, 0.0)$
 $(1.245, 1.934, 1.697, 1.934, true, v_3, 1.0)$

There are three wedges at edge (v_8, v_7) :
 $(0.0, 1.727, 1.934, 0.967, true, v_3, 1.0)$
 $(1.727, 1.824, 1.967, 1.975, true, v_2, 0.0)$
 $(1.824, 1.934, 0.11, 0.0, true, v_7, 1.865)$

Face 3: $f_3 = (v_4, v_0, v_8)$



There are one wedge at edge (v_4, v_0) :

$(0.0, 1.0, 2.778, 1.778, false, v_2, 0.0)$

Due to round off to four decimal places, this windows look like a line.

There are two wedges at edge (v_0, v_8) :

$(0.0, 1.054, 1.778, 2.251, true, v_2, 0.0)$

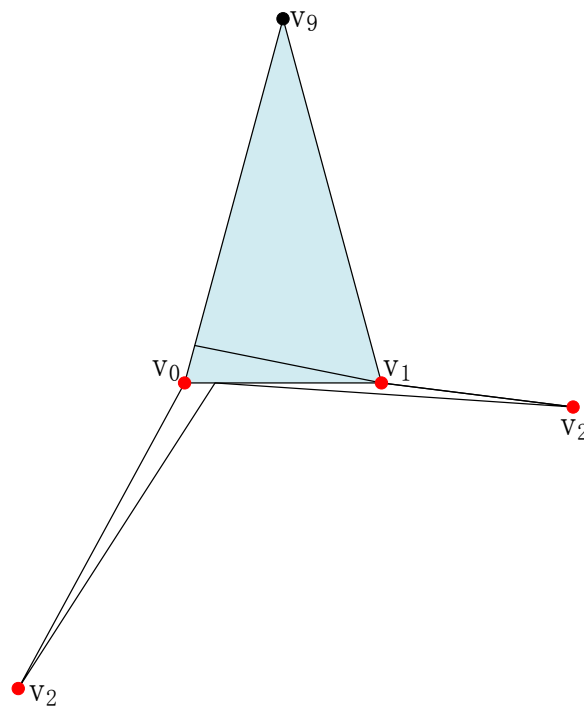
$(1.054, 1.934, 1.251, 1.934, true, v_3, 1.0)$

There are two wedges at edge (v_8, v_4) :

$(0.0, 0.689, 1.934, 1.696, false, v_3, 1.0)$

$(0.689, 1.934, 2.696, 2.778, false, v_2, 0.0)$

Face 4: $f_4 = (v_1, v_0, v_9)$



There are two wedges at edge (v_0, v_1) :

$(0.0, 0.144, 1.778, 1.851, , true, v_2, 0.0)$

$(0.144, 1.0, 1.851, 1.0, true, v_2, 0.0)$

There are one wedge at edge (v_1, v_9) :

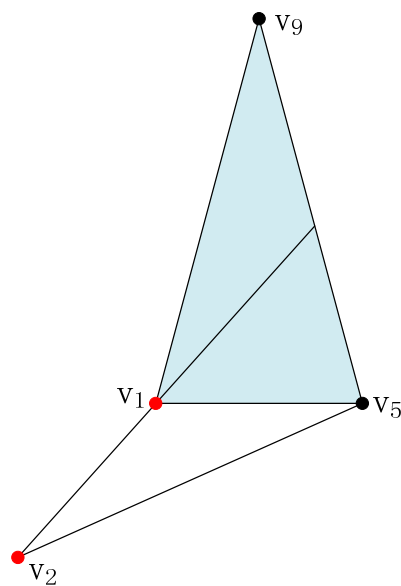
$(0.0, 1.934, 0.0, 1.934, false, v_1, 1.0)$

There are two wedges at edge (v_9, v_0) :

$(0.0, 1.744, 1.934, 0.968, false, v_1, 1.0)$

$(1.744, 1.934, 0.19, 0.0, false, v_0, 1.778)$

Face 5: $f_5 = (v_5, v_1, v_9)$



There are one wedge at edge (v_1, v_5) :

$(0.0, 1.0, 1.0, 1.826, true, v_2, 0.0)$

There are two wedges at edge (v_5, v_9) :

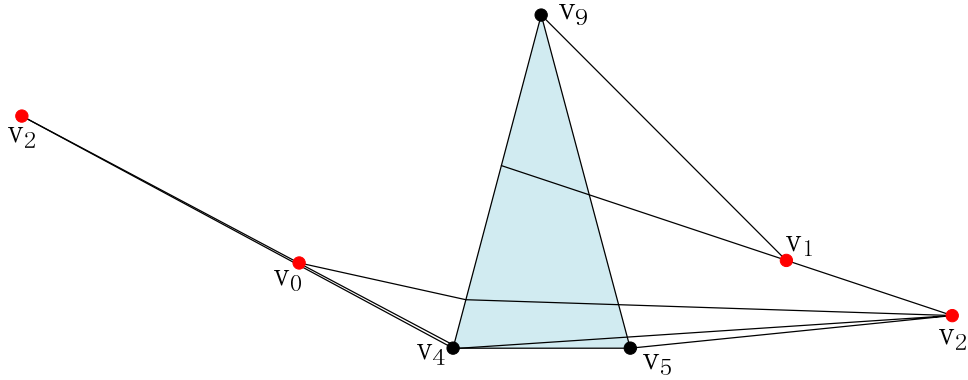
$(0.0, 0.891, 1.826, 2.155, false, v_2, 0.0)$

$(0.891, 1.934, 1.155, 1.934, false, v_1, 1.0)$

There are one wedge at edge (v_9, v_1) :

$(0.0, 1.934, 1.934, 0.0, false, v_1, 1.0)$

Face 6: $f_6 = (v_4, v_5, v_9)$



There are two wedges at edge (v_4, v_5) :

$(0.0, 0.024, 2.778, 2.798, \text{false}, v_2, 0.0)$

$(0.024, 1.0, 2.798, 1.825, \text{false}, v_2, 0.0)$

There are two wedges at edge (v_5, v_9) :

$(0.0, 0.891, 1.826, 2.155, \text{true}, v_2, 0.0)$

$(0.891, 1.934, 1.155, 1.934, \text{true}, v_1, 1.0)$

There are four wedges at edge (v_9, v_4) :

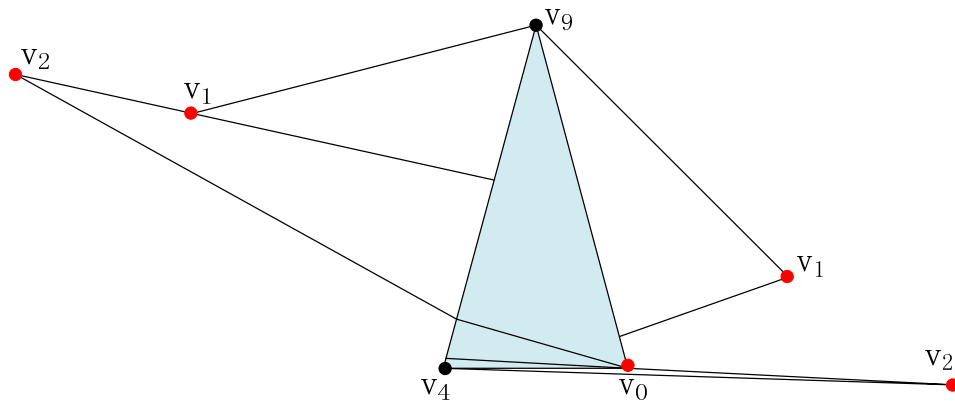
$(0.0, 0.874, 1.934, 1.676, \text{false}, v_1, 1.0)$

$(0.874, 1.659, 2.676, 2.744, \text{false}, v_2, 0.0)$

$(1.659, 1.885, 0.966, 0.988, \text{true}, v_0, 1.778)$

$(1.885, 1.934, 2.766, 2.778, \text{true}, v_2, 0.0)$

Face 7: $f_7 = (v_0, v_4, v_9)$



There are one wedge at edge (v_4, v_0) :

$(0.0, 1.0, 2.778, 1.778, \text{true}, v_2, 0.0)$

There are two wedges at edge (v_0, v_9) :

$(0.0, 0.19, 0.0, 0.19, \text{false}, v_0, 1.778)$

$(0.19, 1.934, 0.968, 1.934, \text{true}, v_1, 1.0)$

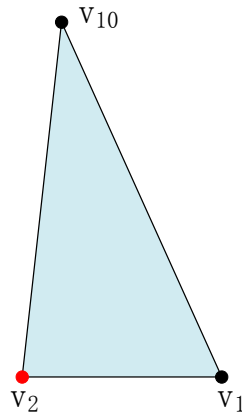
There are four wedges at edge (v_9, v_4) :

$(0.0, 0.874, 1.934, 1.676, \text{true}, v_1, 1.0)$

$(0.874, 1.659, 2.676, 2.744, \text{true}, v_2, 0.0)$

(1.659, 1.885, 0.966, 0.989, *false*, v_0 , 1.778)
(1.885, 1.934, 2.767, 2.778, *false*, v_2 , 0.0)

Face 8: $f_8 = (v_2, v_1, v_{10})$



There are one wedge at edge (v_2, v_1) :

(0.0, 1.0, 0.0, 1.0, *false*, v_2 , 0.0)

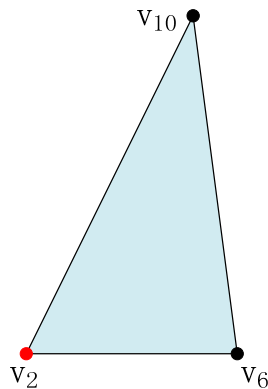
There are one wedge at edge (v_1, v_{10}) :

(0.0, 1.96, 1.0, 1.8, *false*, v_2 , 0.0)

There are one wedge at edge (v_{10}, v_2) :

(0.0, 1.8, 1.8, 0.0, *false*, v_2 , 0.0)

Face 9: $f_9 = (v_6, v_2, v_{10})$



There are one wedge at edge (v_2, v_6) :

(0.0, 1.0, 0.0, 1.0, *false*, v_2 , 0.0)

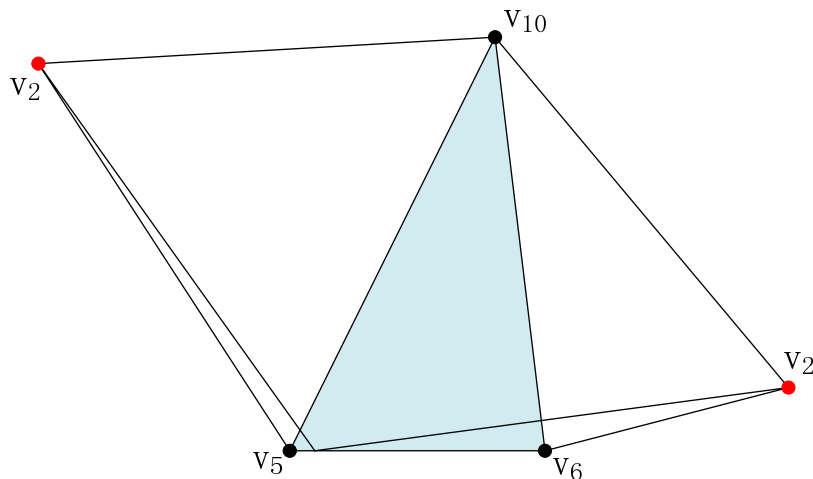
There are one wedge at edge (v_6, v_{10}) :

(0.0, 1.624, 1.0, 1.8, *false*, v_2 , 0.0)

There are one wedge at edge (v_{10}, v_2) :

(0.0, 1.8, 1.8, 0.0, *false*, v_2 , 0.0)

Face 10: $f_{10} = (v_5, v_6, v_{10})$



There are two wedges at edge (v_5, v_6) :

$(0.0, 0.102, 1.826, 1.883, false, v_2, 0.0)$

$(0.102, 1.0, 1.833, 1.0, false, v_2, 0.0)$

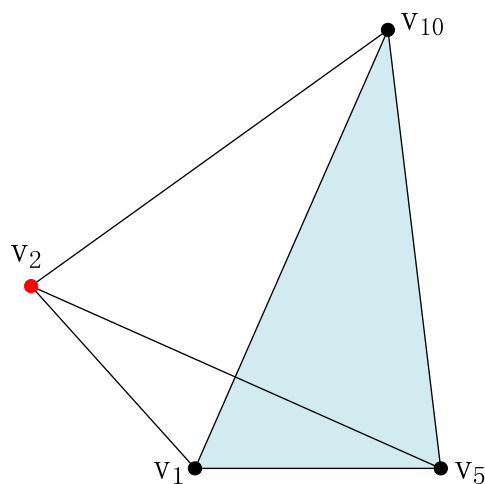
There are one wedge at edge (v_6, v_{10}) :

$(0.0, 1.624, 1.0, 1.8, true, v_2, 0.0)$

There are one wedge at edge (v_{10}, v_5) :

$(0.0, 1.8, 1.8, 1.826, true, v_2, 0.0)$

Face 11: $f_{11} = (v_1, v_5, v_{10})$



There are one wedge at edge (v_1, v_5) :

$(0.0, 1.0, 1.0, 1.826, false, v_2, 0.0)$

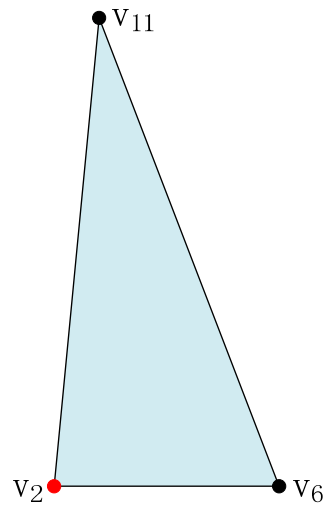
There are one wedge at edge (v_5, v_{10}) :

$(0.0, 1.8, 1.826, 1.8, false, v_2, 0.0)$

There are one wedge at edge (v_{10}, v_1) :

$(0.0, 1.96, 1.8, 1.0, true, v_2, 0.0)$

Face 12: $f_{12} = (v_2, v_6, v_{11})$



There are one wedge at edge (v_2, v_6) :

$(0.0, 1.0, 0.0, 1.0, false, v_2, 0.0)$

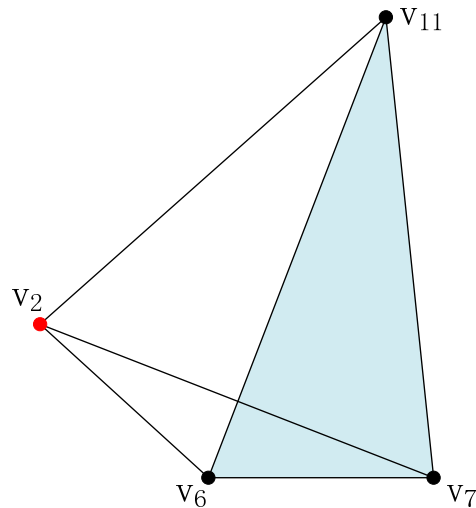
There are one wedge at edge (v_6, v_{11}) :

$(0.0, 2.211, 1.0, 2.071, false, v_2, 0.0)$

There are one wedge at edge (v_{11}, v_2) :

$(0.0, 2.071, 2.071, 0.0, false, v_2, 0.0)$

Face 13: $f_{13} = (v_6, v_7, v_{11})$



There are one wedge at edge (v_6, v_7) :

$(0.0, 1.0, 1.0, 1.865, false, v_2, 0.0)$

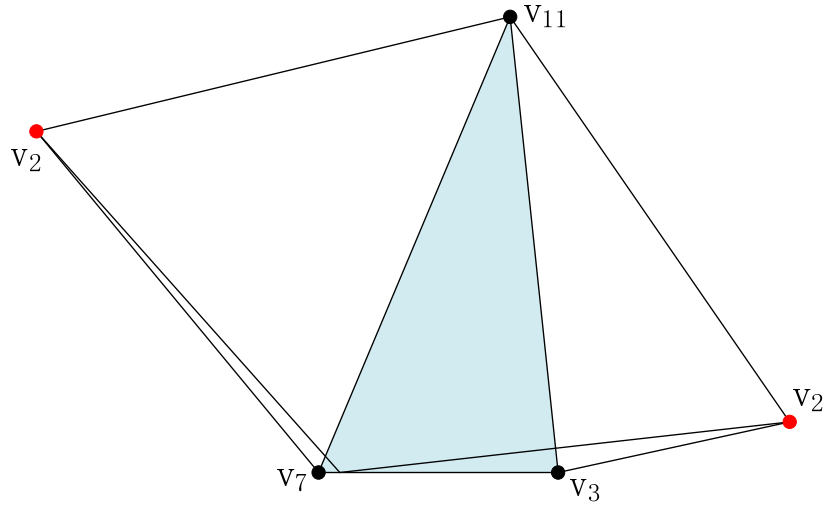
There are one wedge at edge (v_7, v_{11}) :

$(0.0, 2.071, 1.865, 2.071, false, v_2, 0.0)$

There are one wedge at edge (v_{11}, v_6) :

$(0.0, 2.211, 2.071, 1.0, true, v_2, 0.0)$

Face 14: $f_{14} = (v_7, v_3, v_{11})$



There are two wedges at edge (v_7, v_3) :

$(0.0, 0.075, 1.865, 1.914, false, v_2, 0.0)$

$(0.075, 1.0, 1.914, 1.0, false, v_2, 0.0)$

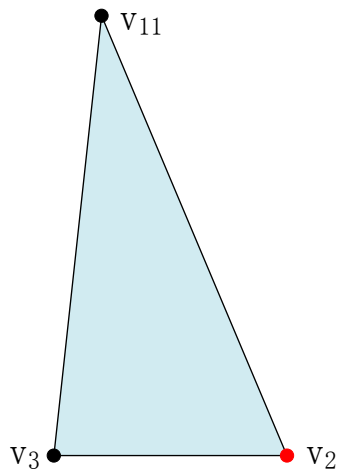
There are one wedge at edge (v_3, v_{11}) :

$(0.0, 1.921, 1.0, 2.071, true, v_2, 0.0)$

There are one wedge at edge (v_{11}, v_7) :

$(0.0, 2.071, 2.071, 1.865, true, v_2, 0.0)$

Face 15: $f_{15} = (v_3, v_2, v_{11})$



There are one wedge at edge (v_3, v_2) :

$(0.0, 1.0, 1.0, 0.0, false, v_2, 0.0)$

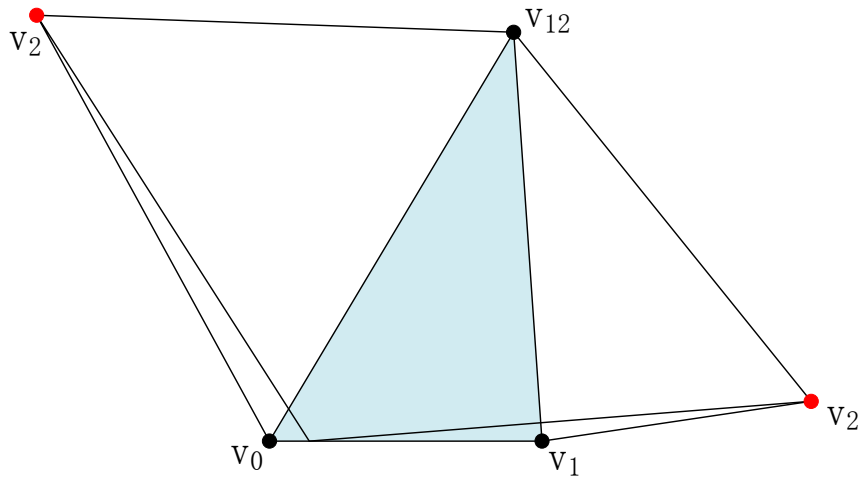
There are one wedge at edge (v_2, v_{11}) :

$(0.0, 2.071, 0.0, 0.071, false, v_2, 0.0)$

There are one wedge at edge (v_{11}, v_3) :

$(0.0, 1.921, 2.071, 1.0, false, v_2, 0.0)$

Face 16: $f_{16} = (v_0, v_1, v_{12})$



There are two wedges at edge (v_0, v_1) :

$(0.0, 0.144, 1.778, 1.851, false, v_2, 0.0)$

$(0.144, 1.0, 1.851, 1.0, false, v_2, 0.0)$

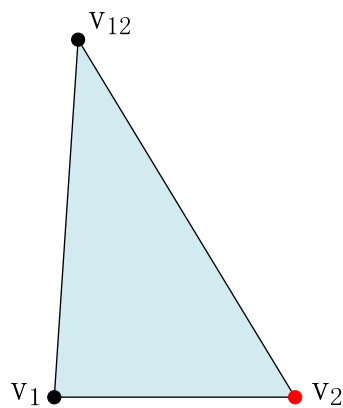
There are one wedge at edge (v_1, v_{12}) :

$(0.0, 1.507, 1.0, 1.752, true, v_2, 0.0)$

There are one wedge at edge (v_{12}, v_0) :

$(0.0, 1.752, 1.752, 1.778, true, v_2, 0.0)$

Face 17: $f_{17} = (v_1, v_2, v_{12})$



There are one wedge at edge (v_1, v_2) :

$(0.0, 1.0, 1.0, 0.0, false, v_2, 0.0)$

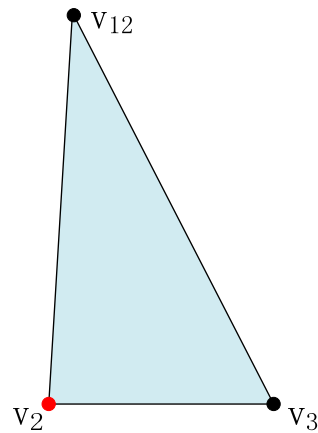
There are one wedge at edge (v_2, v_{12}) :

$(0.0, 1.752, 0.0, 1.752, false, v_2, 0.0)$

There are one wedge at edge (v_{12}, v_1) :

$(0.0, 1.507, 1.752, 1.0, false, v_2, 0.0)$

Face 18: $f_{18} = (v_2, v_3, v_{12})$



There are one wedge at edge (v_2, v_3) :

$(0.0, 1.0, 0.0, 1.0, false, v_2, 0.0)$

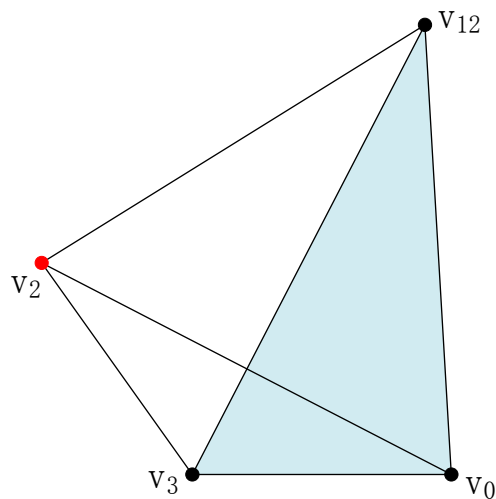
There are one wedge at edge (v_3, v_{12}) :

$(0.0, 1.967, 1.0, 1.752, false, v_2, 0.0)$

There are one wedge at edge (v_{12}, v_2) :

$(0.0, 1.752, 1.752, 0.0, false, v_2, 0.0)$

Face 19: $f_{19} = (v_3, v_0, v_{12})$



There are one wedge at edge (v_3, v_0) :

$(0.0, 1.0, 1.0, 1.778, false, v_2, 0.0)$

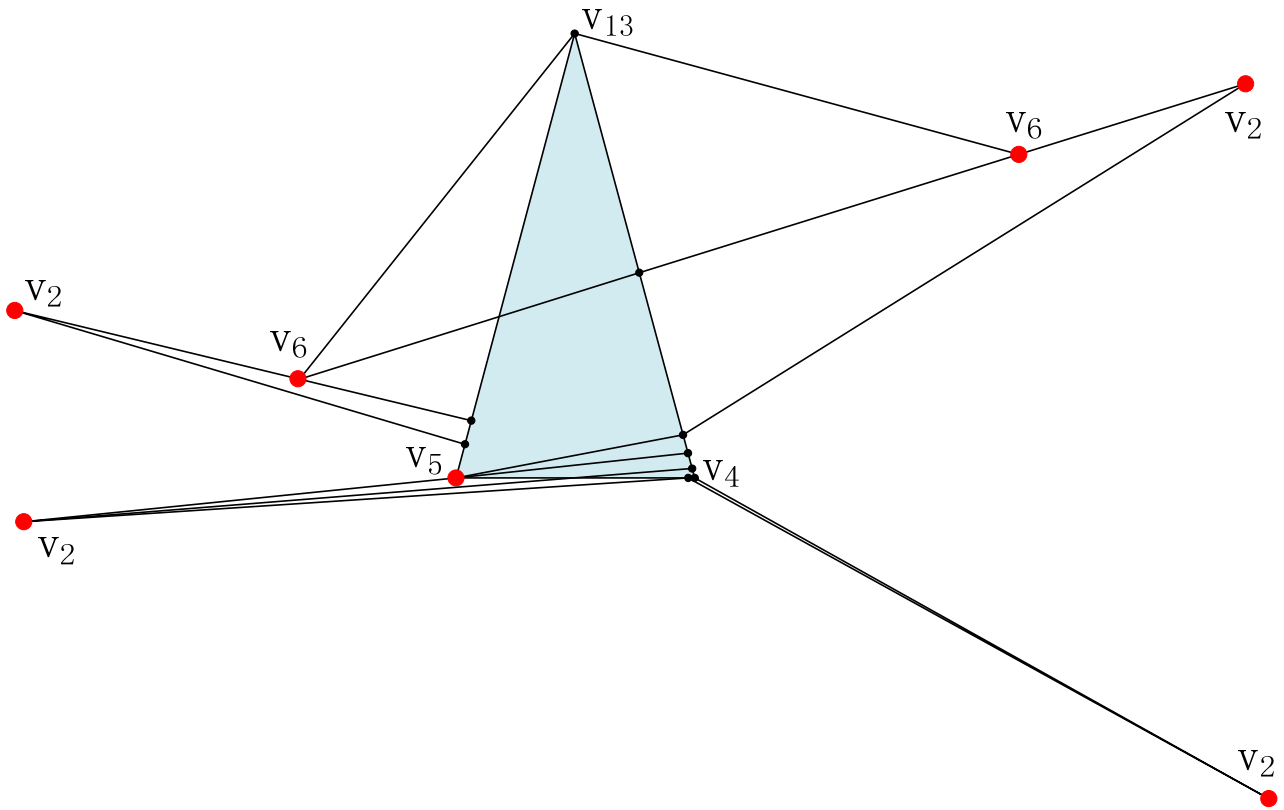
There are one wedge at edge (v_0, v_{12}) :

$(0.0, 1.752, 1.778, 1.752, false, v_2, 0.0)$

There are one wedge at edge (v_{12}, v_3) :

$(0.0, 1.967, 1.752, 1.0, true, v_2, 0.0)$

Face 20: $f_{20} = (v_5, v_4, v_{13})$



There are two wedges at edge (v_5, v_4) :

- $(0.0, 0.976, 1.826, 2.799, \text{true}, v_2, 0.0)$
- $(0.976, 1.0, 2.799, 2.778, \text{true}, v_2, 0.0)$

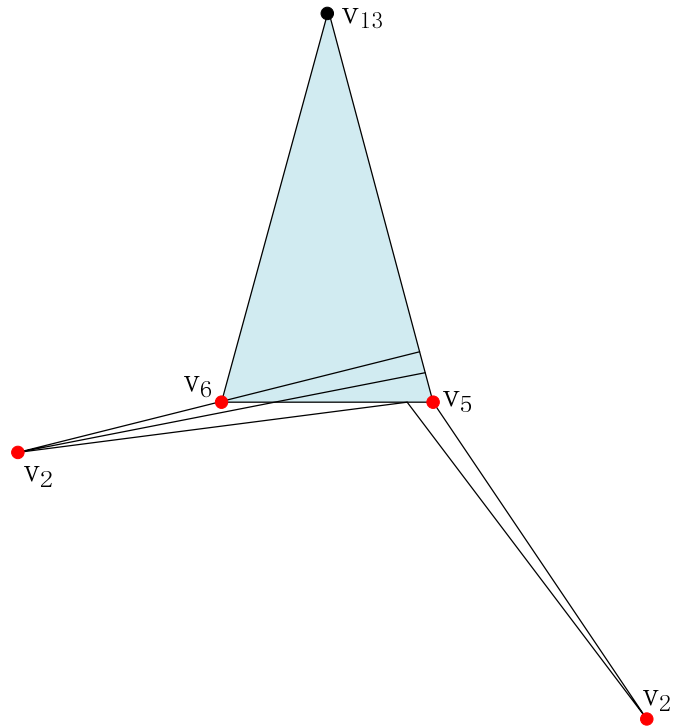
There are five wedges at edge (v_4, v_{13}) :

- $(0.0, 0.038, 0.0, 0.038, \text{false}, v_4, 2.778)$
- $(0.038, 0.104, 2.816, 2.804, \text{false}, v_2, 0.0)$
- $(0.104, 0.188, 0.978, 0.968, \text{false}, v_5, 1.826)$
- $(0.188, 0.891, 2.794, 2.675, \text{true}, v_2, 0.0)$
- $(0.891, 1.934, 1.675, 1.934, \text{true}, v_6, 1.0)$

There are three wedges at edge (v_{13}, v_5) :

- $(0.0, 1.690, 1.934, 0.966, \text{true}, v_6, 1.0)$
- $(1.690, 1.790, 1.966, 1.970, \text{true}, v_2, 0.0)$
- $(1.790, 1.934, 0.144, 0.0, \text{false}, v_5, 1.826)$

Face 21: $f_{21} = (v_6, v_5, v_{13})$



There are two wedges at edge (v_6, v_5) :

$(0.0, 0.898, 1.0, 1.884, \text{true}, v_2, 0.0)$

$(0.898, 1.0, 1.884, 1.826, \text{true}, v_2, 0.0)$

There are three wedges at edge (v_5, v_{13}) :

$(0.0, 0.144, 0.0, 0.144, \text{false}, v_5, 1.826)$

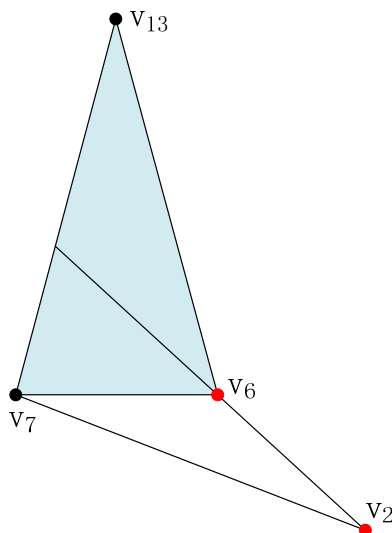
$(0.144, 0.244, 1.970, 1.966, \text{false}, v_2, 0.0)$

$(0.244, 1.934, 0.966, 1.934, \text{false}, v_6, 1.0)$

There are one wedge at edge (v_{13}, v_6) :

$(0.0, 1.934, 1.934, 0.0, \text{false}, v_6, 1.0)$

Face 22: $f_{22} = (v_7, v_6, v_{13})$



There are one wedge at edge (v_7, v_6) :

$(0.0, 1.0, 1.865, 1.0, true, v_2, 0.0)$

There are one wedge at edge (v_6, v_{13}) :

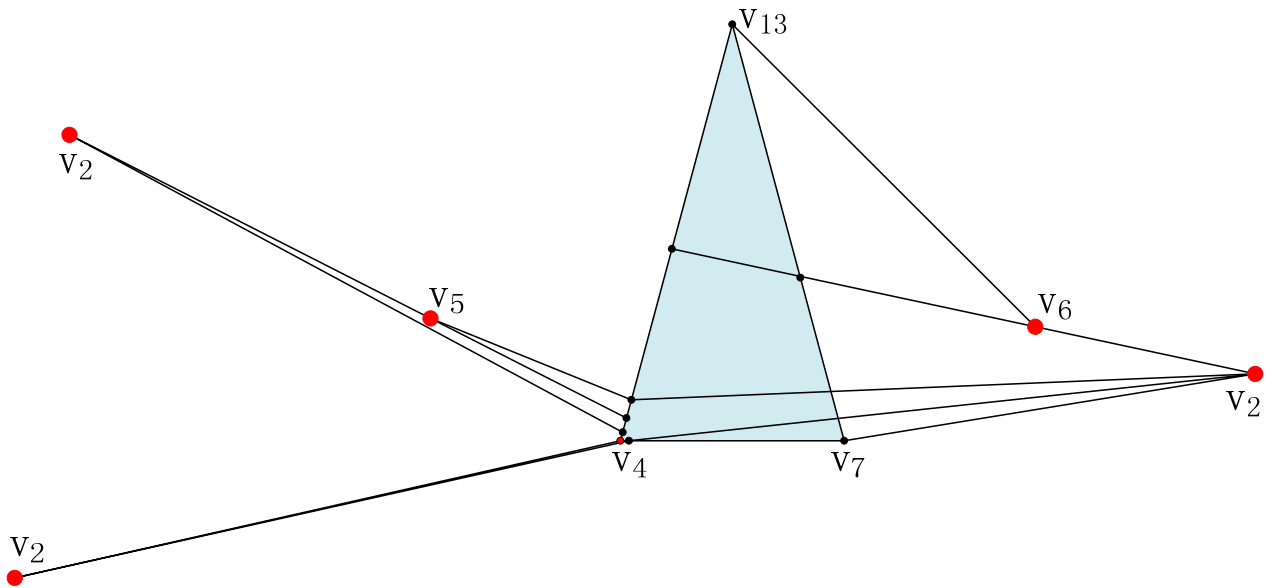
$(0.0, 1.934, 0.0, 1.934, false, v_6, 1.0)$

There are two wedges at edge (v_{13}, v_7) :

$(0.0, 1.174, 1.934, 1.088, false, v_6, 1.0)$

$(1.174, 1.934, 2.088, 1.865, false, v_2, 0.0)$

Face 23: $f_{23} = (v_4, v_7, v_{13})$.



There are two wedges at edge (v_4, v_7) :

$(0.0, 0.040, 2.778, 2.817, true, v_2, 0.0)$

$(0.040, 1.0, 2.817, 1.865, false, v_2, 0.0)$

There are two wedges at edge (v_7, v_{13}) :

$(0.0, 0.760, 1.865, 2.088, true, v_2, 0.0)$

$(0.760, 1.934, 1.088, 1.934, true, v_6, 1.0)$

There are five wedges at edge (v_{13}, v_4) :

$(0.0, 1.043, 1.934, 1.675, false, v_6, 1.0)$

$(1.043, 1.746, 2.675, 2.794, false, v_2, 0.0)$

$(1.746, 1.830, 0.968, 0.978, true, v_5, 1.826)$

$(1.830, 1.896, 2.804, 2.816, true, v_2, 0.0)$

$(1.896, 1.934, 0.038, 0.0, false, v_4, 2.778)$