Supplemental Material for 'Computational Design of Transforming Pop-up Books'

NAN XIAO and ZHE ZHU
TNList, Department of Computer Science and Technology, Tsinghua University
and
RALPH R. MARTIN
School of Computer Science and Informatics, Cardiff University
and
KUN XU and JIA-MING LU and SHI-MIN HU
TNList, Department of Computer Science and Technology, Tsinghua University

Degrees of Freedom of a Mechanism

In mechanics, the number of degrees of freedom (DoF) of a mechanism is the number of independent relative motions within a set of connected rigid bodies. In the following, we first prove that each of the three basic mechanisms we use have one DoF. Then, we prove that the whole popup mechanism has one DoF, by proving that the allowable combinations of the basic mechanisms also have one DoF.

Theorem 1: Kutzbach-Gruebler Equation The number M of degrees of freedom of a mechanism composed of n moving parts and j joints is given by:

$$M = 3(n - j) + \sum_{i=1}^{j} f_i,$$

where f_i denotes the number of degrees of freedom of the i-th joint.

Proof See [McCarthy and Soh 2011].

Proposition 1 *A parallel mechanism composed of four quadrilateral patches T_i, $1 \leq i \leq 4$ connected cyclically by four mutually parallel hinges, and in which patches T_1 and T_3, and T_2 and T_4, are parallel in pairs, has 1 DoF.*

Proof Such a parallel mechanism has 3 moving patches and 4 hinges. Each hinge has one DOF. Thus by Theorem 1, a parallel mechanism has $M_p = 3 \times (3 - 4) + 4 \times 1 = 1$ DoF.

Proposition 2 *A V-fold mechanism composed of four patches with four hinges, where the extended lines of the four hinges intersect at one point, and the angles α_{ij} between hinges h_i and h_j satisfy $\alpha_{14} = \alpha_{23}$ and $\alpha_{12} = \alpha_{34}$, has 1 DoF.*

Proof A V-fold mechanism also has 3 moving patches and 4 hinges, each with 1 DoF. Thus a V-fold mechanism again has $M_v = 3 \times (3 - 4) + 4 \times 1 = 1$ DoF.

Proposition 3 *A slide mechanism composed of one rectangular slide patch, a hinge and a slit associated with a parallel mechanism or a V-fold mechanism (constrained as defined in the paper) has 1 DoF.*

Proof In a slide mechanism, a virtual constraint exists between the slide patch and the slit, so it forms an RRPR linkage. Thus, it has 3 moving patches (apart from the slide patch) and each has 3 degrees of freedom [McCarthy and Soh 2011]. Thus a slide mechanism has $M_s = 3 \times (3 - 4) + 4 \times 1 = 1$ DoF.

Proposition 4 *Any mechanism which is the combination of these three basic mechanisms also has 1 DoF.*

Proof Let some combined mechanism containing i basic mechanisms have M^i DoF. Suppose we add one more basic mechanism to it, using the hinge linkage introduced in Section 3.3, giving a new mechanism with M^{i+1} DoF. Since all of the basic mechanisms are four-bar linkages, the way in which the newly added basic mechanism is combined with the existing mechanism is always the same: altogether, two patches and three hinges are added.

According to the Kutzbach-Gruebler Equation, the new mechanism has $M^{i+1} = M^i + \Delta M = M^i + 3 \times (2 - 3) + 3 \times 1 = M^i + 0 = M^i$ DoF. We must start with a single mechanism, and as noted, each of the single mechanisms has 1 DoF. Thus, by induction, a mechanism formed from any number of basic mechanisms has 1 DoF.

Table I. : For each example, we give numbers of each kind of basic mechanism, and extended patches, used. PM: parallel mechanism, VM: V-fold mechanism, SM: slide mechanism, EP: extended patch.

Example Name	Figure	PM	VM	SM	EP
frog-19 to bell-3	1	1	4	0	8
tree-14 to flatfish-1	2	0	4	0	5
chicken-8 to personal_car-2	3	0	4	0	7
flatfish-12 to jar-3	4	0	3	0	5
shoe-16 to dog-20	5	2	3	0	8
face-8 to hcircle-18	6	2	2	0	7
cellular_phone-2 to key-11	7	1	1	0	4
device4-18 to ray-5	8	1	3	1	7
teddy-20 to device1-3	9	1	3	0	8
turtle-18 to hat-3	10	1	2	0	5

Additional examples

We give 10 further results of our algorithm here. We randomly selected 10 pairs of shapes from the MPEG7 dataset [MPE] as inputs to our algorithm. We show the output results below as well as giving detailed statistics in Table I.

REFERENCES

MPEG7 shape dataset. http://www.dabi.temple.edu/~shape/MPEG7/dataset.html.

McCarthy, J. M. and Soh, G. S. 2011. *Geometric Design of Linkages*. Vol. 11. Springer-Verlag New York.

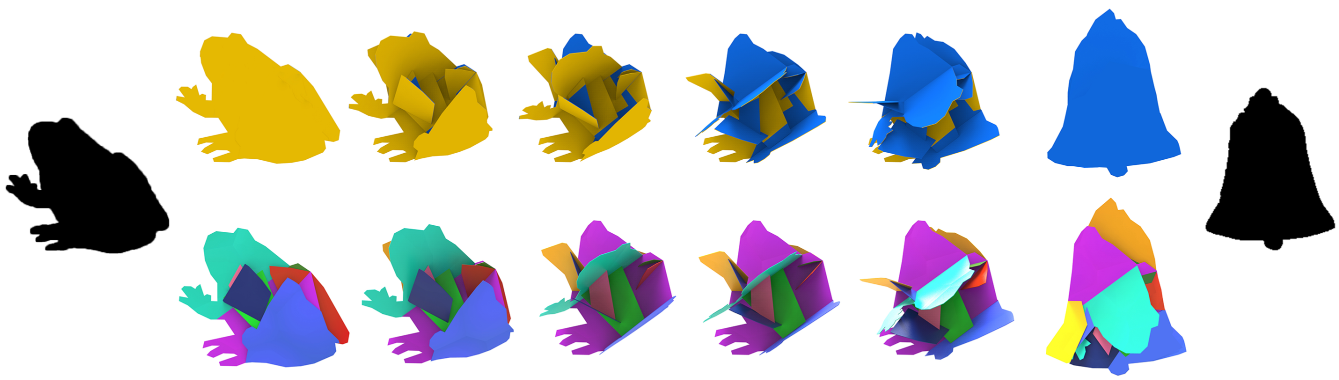

Fig. 1: From frog-19 to bell-3.

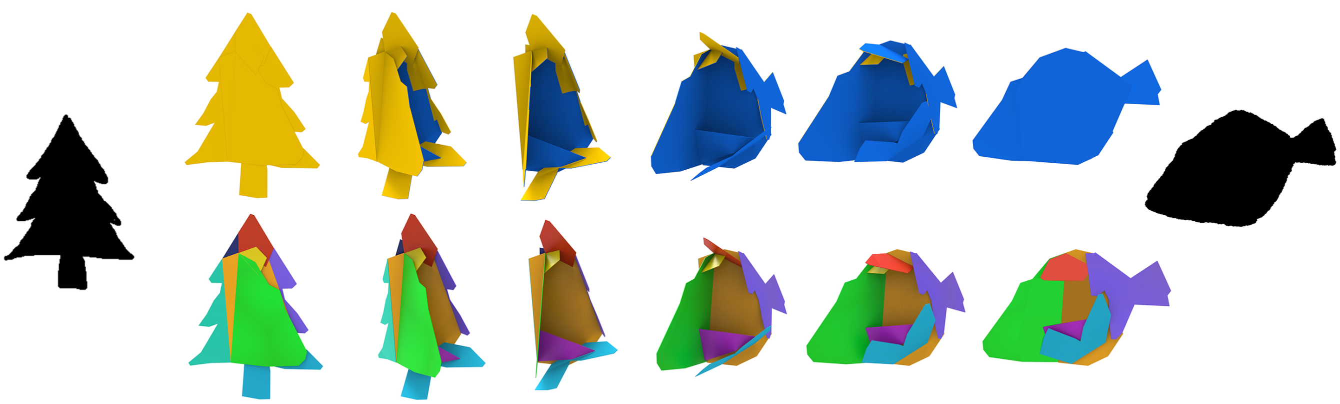

Fig. 2: From tree-14 to flatfish-1.

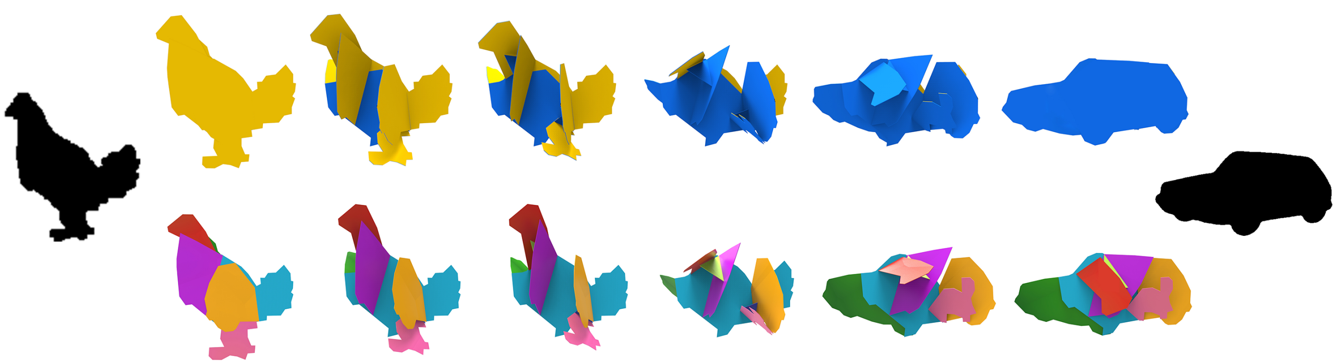

Fig. 3: From chicken-8 to personal_car-2.

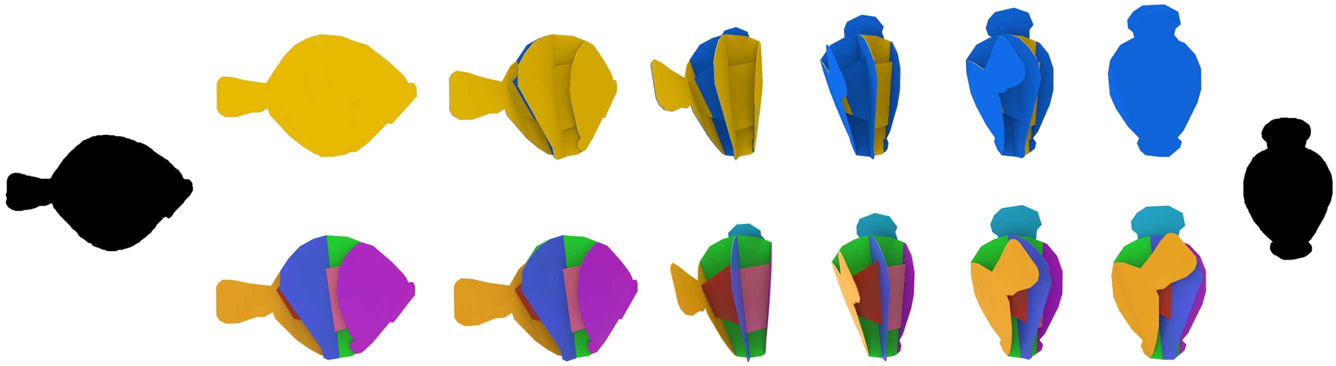

Fig. 4: From flatfish-12 to jar-3.

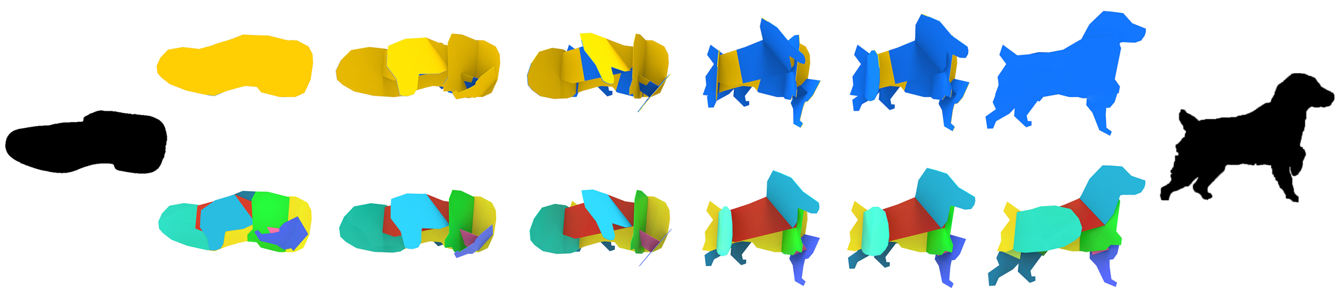

Fig. 5: From shoe-16 to dog-20.

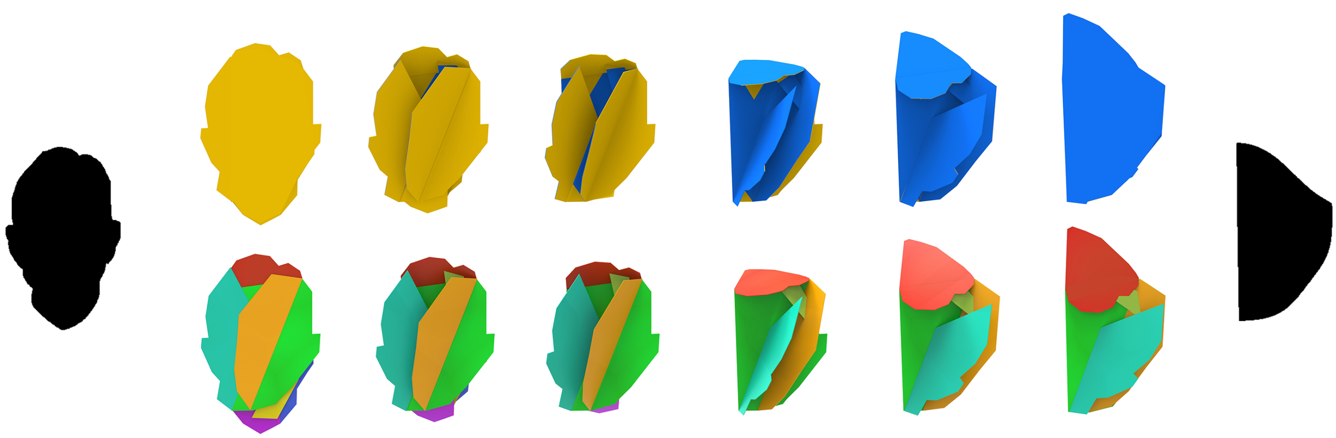

Fig. 6: From face-8 to hcircle-18.

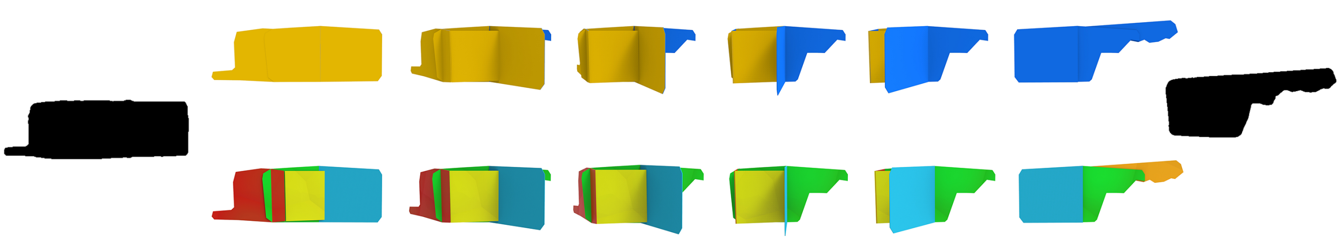

Fig. 7: From cellular_phone-2 to key-11.

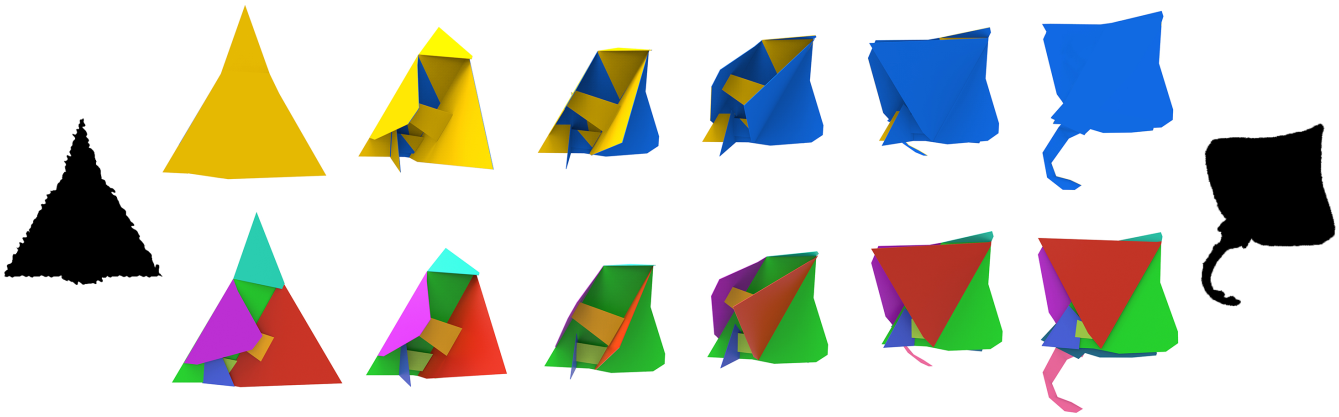

Fig. 8: From device4-18 to ray-5.

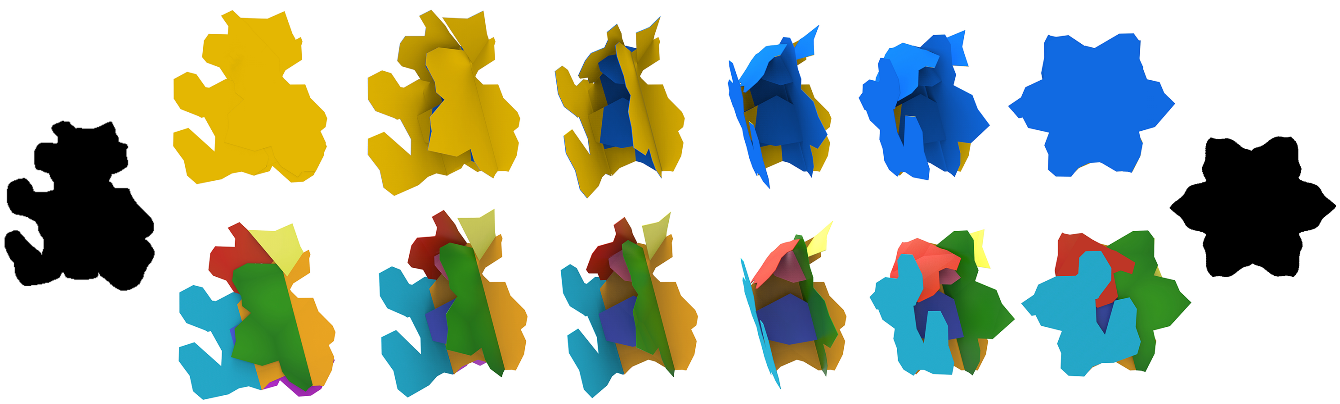

Fig. 9: From teddy-20 to device1-3.

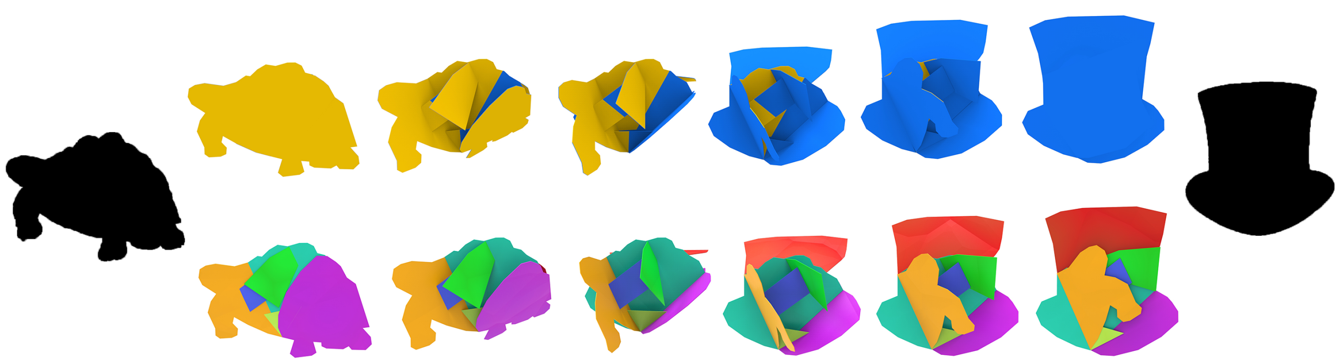

Fig. 10: From turtle-18 to hat-3.