Three Dimensional Strain Field Morphing

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Morphing

- What is Morphing?
- Why some morphing results are natural while others not?
  - The key is the deformation process should be smooth and uniform.
- Problem: What is *Deformation*? How to define it in mathematics?

Deformation Definition?

- Other morphing methods
  - Have not given deformation definition before.
- In mechanics, a concept for describing shape deformation has been used for hundreds of years.
- That is:
  - ***Strain***

Strain

- What is ***Strain***?
  - Strain is a tensor quantity that describes shape deformation extent at each point.
  - Strain is a microscopic quantity.
  - In 3D, strain has 6 independent components.

Strain

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left[\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 + \left(\frac{\partial z'}{\partial x}\right)^2 - 1\right] \\ \frac{1}{2}\left[\left(\frac{\partial x'}{\partial y}\right)^2 + \left(\frac{\partial y'}{\partial y}\right)^2 + \left(\frac{\partial z'}{\partial y}\right)^2 - 1\right] \\ \frac{1}{2}\left[\left(\frac{\partial x'}{\partial z}\right)^2 + \left(\frac{\partial y'}{\partial z}\right)^2 + \left(\frac{\partial z'}{\partial z}\right)^2 - 1\right] \\ \frac{\partial x'}{\partial y}\frac{\partial x'}{\partial z} + \frac{\partial y'}{\partial y}\frac{\partial y'}{\partial z} + \frac{\partial z'}{\partial y}\frac{\partial z'}{\partial z} \\ \frac{\partial x'}{\partial z}\frac{\partial x'}{\partial x} + \frac{\partial y'}{\partial z}\frac{\partial y'}{\partial x} + \frac{\partial z'}{\partial z}\frac{\partial z'}{\partial x} \\ \frac{\partial x'}{\partial x}\frac{\partial x'}{\partial y} + \frac{\partial y'}{\partial x}\frac{\partial y'}{\partial y} + \frac{\partial z'}{\partial x}\frac{\partial z'}{\partial y} \end{bmatrix}
$$

Strain

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix}
\frac{1}{2}\left[\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 + \left(\frac{\partial z'}{\partial x}\right)^2 - 1\right] \\
\frac{1}{2}\left[\left(\frac{\partial x'}{\partial y}\right)^2 + \left(\frac{\partial y'}{\partial y}\right)^2 + \left(\frac{\partial z'}{\partial y}\right)^2 - 1\right] \\
\frac{1}{2}\left[\left(\frac{\partial x'}{\partial z}\right)^2 + \left(\frac{\partial y'}{\partial z}\right)^2 + \left(\frac{\partial z'}{\partial z}\right)^2 - 1\right] \\
\frac{\partial x'}{\partial y}\frac{\partial x'}{\partial z} + \frac{\partial y'}{\partial y}\frac{\partial y'}{\partial z} + \frac{\partial z'}{\partial y}\frac{\partial z'}{\partial z} \\
\frac{\partial x'}{\partial z}\frac{\partial x'}{\partial x} + \frac{\partial y'}{\partial z}\frac{\partial y'}{\partial x} + \frac{\partial z'}{\partial z}\frac{\partial z'}{\partial x} \\
\frac{\partial x'}{\partial x}\frac{\partial x'}{\partial y} + \frac{\partial y'}{\partial x}\frac{\partial y'}{\partial y} + \frac{\partial z'}{\partial x}\frac{\partial z'}{\partial y}
\end{bmatrix}
$$

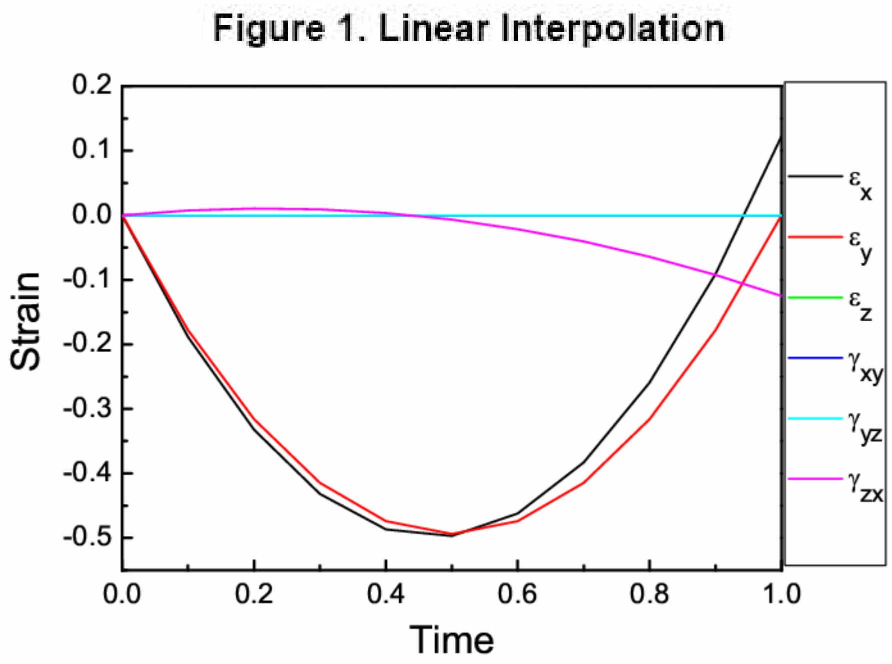
ε_i gives the local infinitesimal scaling along the i axis direction at point p.

Strain

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left[\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 + \left(\frac{\partial z'}{\partial x}\right)^2 - 1\right] \\ \frac{1}{2}\left[\left(\frac{\partial x'}{\partial y}\right)^2 + \left(\frac{\partial y'}{\partial y}\right)^2 + \left(\frac{\partial z'}{\partial y}\right)^2 - 1\right] \\ \frac{1}{2}\left[\left(\frac{\partial x'}{\partial z}\right)^2 + \left(\frac{\partial y'}{\partial z}\right)^2 + \left(\frac{\partial z'}{\partial z}\right)^2 - 1\right] \\ \frac{\partial x'}{\partial y}\frac{\partial x'}{\partial z} + \frac{\partial y'}{\partial y}\frac{\partial y'}{\partial z} + \frac{\partial z'}{\partial y}\frac{\partial z'}{\partial z} \\ \frac{\partial x'}{\partial z}\frac{\partial x'}{\partial x} + \frac{\partial y'}{\partial z}\frac{\partial y'}{\partial x} + \frac{\partial z'}{\partial z}\frac{\partial z'}{\partial x} \\ \frac{\partial x'}{\partial x}\frac{\partial x'}{\partial y} + \frac{\partial y'}{\partial x}\frac{\partial y'}{\partial y} + \frac{\partial z'}{\partial x}\frac{\partial z'}{\partial y} \end{bmatrix}
$$

γ_{ij} represents the relative change in angle between lines initially in the i and j directions at point p.

Deformation Analysis with Strain



Strain in Morphing

- Strain is a quantitative tool in analyzing shape deformation.
- If we want the morphing process is deformation uniform, the strain change should be monotone.

3D Strain Field Morphing

- Input:
  - Source and target meshes
  - consistent 3D tetrahedron meshes.
- Continuous
  - Assume all shapes all continuous model; they are only be approximately represented by discrete tetrahedron meshes.

Two fields

- Position field
  - All points position in the continuous model construct a position field.
- Strain field
  - Field to describe shape deformation between two shapes, which can be calculated from two position field.

3D Strain Field Morphing

- Steps
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field

3D Strain Field Morphing

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  - Strain field calculation
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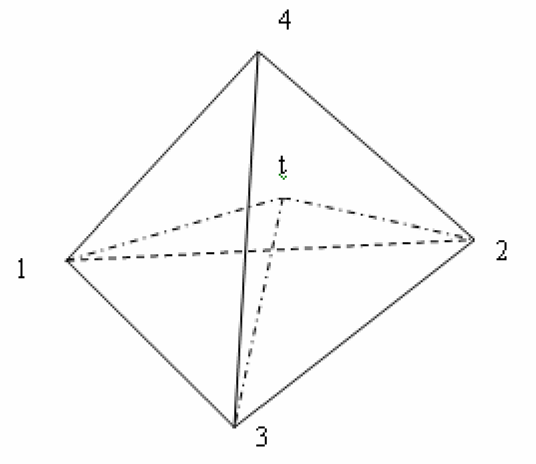
Strain field calculation

- Finite element theory is used in strain field calculation.
- Linear tetrahedron element is used.



$$x = \sum N_i x_i$$

$$y = \sum N_i y_i$$

$$z = \sum N_i z_i$$

3D Strain Field Morphing

- Steps
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field

Strain field interpolation

- For small deformation, linear strain field interpolation can be used.

$$\varepsilon^{t} = (1 - t)\varepsilon^{n}$$

- For large deformation, modified strain field interpolation is used.

Strain field interpolation

- Modified strain field interpolation

$$\varepsilon^{t}{}_{x} = \frac{1}{2}\left[\left[t\sqrt{1+2\varepsilon_{x}{}^{n}} + (1-t)\right]^{2} - 1\right]$$

$$\varepsilon^{t}{}_{y} = \frac{1}{2}\left[\left[t\sqrt{1+2\varepsilon_{y}{}^{n}} + (1-t)\right]^{2} - 1\right]$$

$$\varepsilon^{t}{}_{z} = \frac{1}{2}\left[\left[t\sqrt{1+2\varepsilon_{z}{}^{n}} + (1-t)\right]^{2} - 1\right]$$

$$\gamma^{t}_{yz} = (1-t)\gamma^{n}_{yz}$$

$$\gamma^{t}_{zx} = (1-t)\gamma^{n}_{zx}$$

$$\gamma^{t}_{xy} = (1-t)\gamma^{n}_{xy}$$

3D Strain Field Morphing

- Steps
  - Strain field calculation
  - Strain field interpolation
  - Calculate position field from strain field

Calculation Position field

- Find the position field which has nearest strain field to the interpolated strain field.

$$W = \sum_{i=0}^{n-1} \int \left(\varepsilon''^{t} - \varepsilon^{t}\right)^{T} \left(\varepsilon''^{t} - \varepsilon^{t}\right) d\Omega$$

- Solver
  - Gradient optimization method.
  - Nonlinear equation solver(Newton-Raphson method).

Results

Elephant

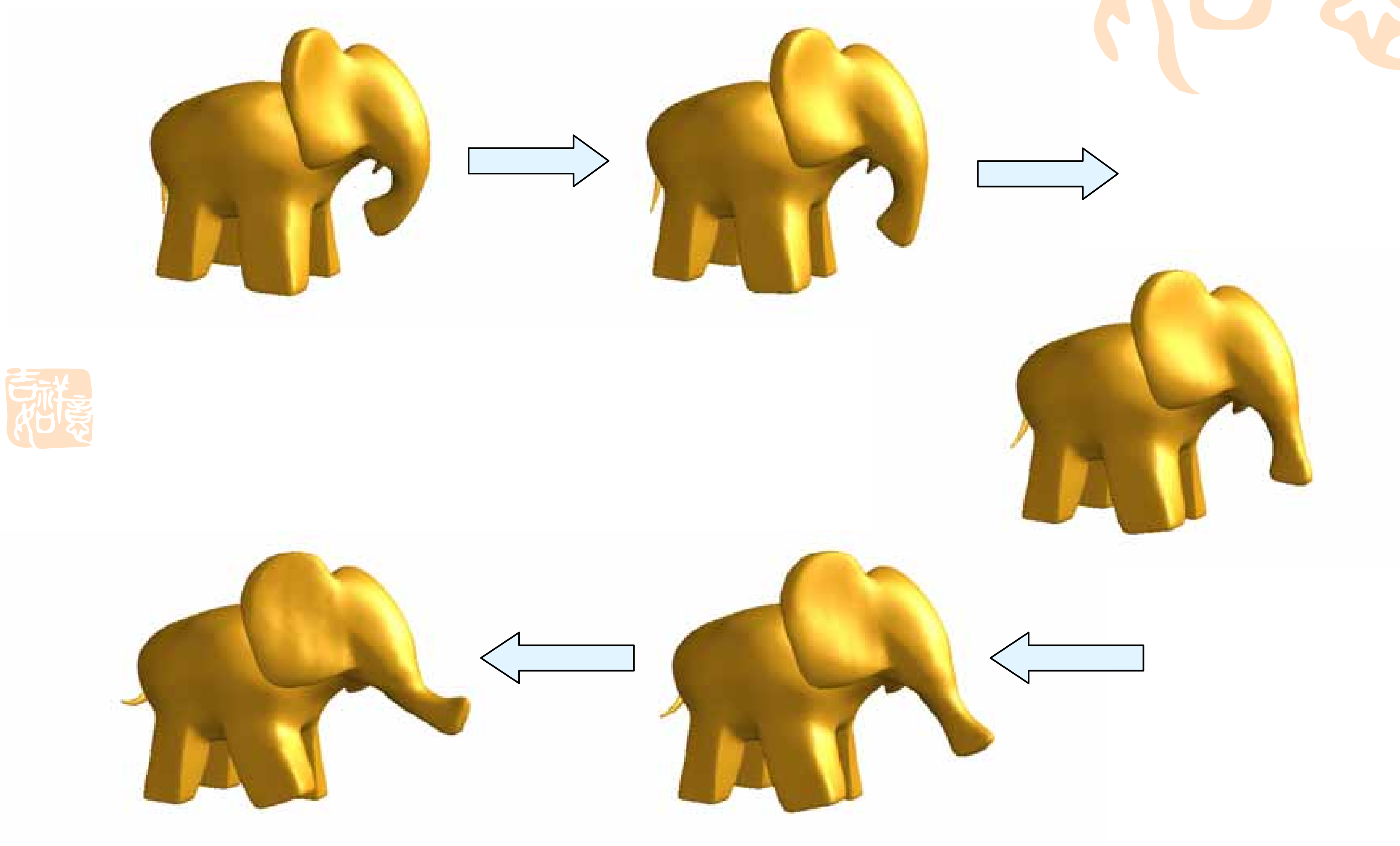
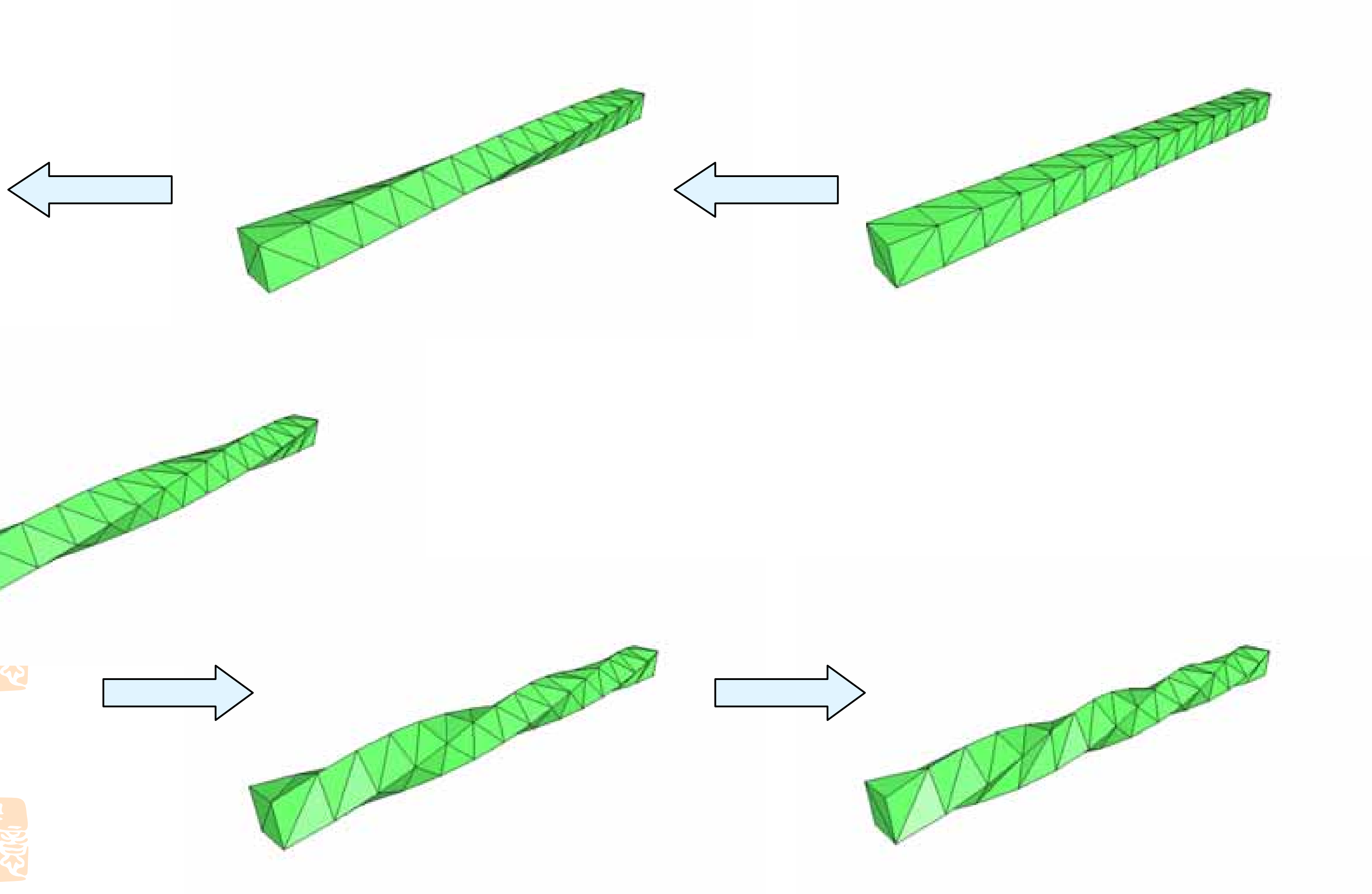


Figure 1. Linear Interpolation

Twist



Demo

Helix

Demo

Animal Morphing

Question?

Thanks!