Morphing Based on Strain Field Interpolation

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Introduction

- In recent years, morphing has been studied both on image based and geometry based ways. In general, geometry based techniques can handle objects with more complex shapes and larger deformation.
- Work which generate excited results was presented by Alexa et al. on Siggraph 2000.

- The aim of morphing is to generate a smooth transforming process that change one object to another, both in shape and texture.

- For geometry based method, the key is to generate a smooth shape changing process, i.e., the deformation process of object should be uniform.
- Deformation is a infinitesimal concept other than a macroscopic quantity.
- In fact, physics has provided us a strong tool to analyze object deformation–Strain.

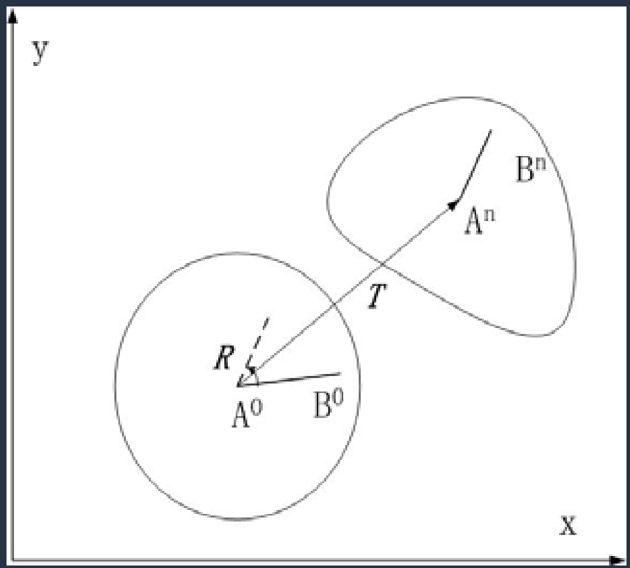

Figure 1

As in figure 1, an object motion always can be decomposed into three parts: Translation, Rotation and Deformation. Translation and Rotation are all rigid body motions.

- If only rigid body motion happens, very good results can be got by simply linear interpolation. What is tricky is how to treat the deformation.
- For morphing problem, the object deformation is always very large, so large deformation strain formula should be used instead of engineering strain formula often used.

- The strain field of large deformation is given by:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left[\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 - 1\right] \\ \frac{1}{2}\left[\left(\frac{\partial x'}{\partial y}\right)^2 + \left(\frac{\partial y'}{\partial y}\right)^2 - 1\right] \\ \frac{\partial x'}{\partial x}\frac{\partial x'}{y} + \frac{\partial y'}{\partial x}\frac{\partial y'}{y} \end{bmatrix}$$

Eqn.1

ε_x gives the local infinitesimal scaling in the *x* direction, ε_y gives the scaling in the *y* direction, γ_{xy} is the shear strain which represents the relative change in angle between lines initially in the *x* and *y* directions at *p*.

ε_x Is positive when the point is in tension along the *x* direction, and negative in compression; similarly for ε_y.

γ_{xy} is positive when the angle becomes smaller, and negative when the angle becomes larger.

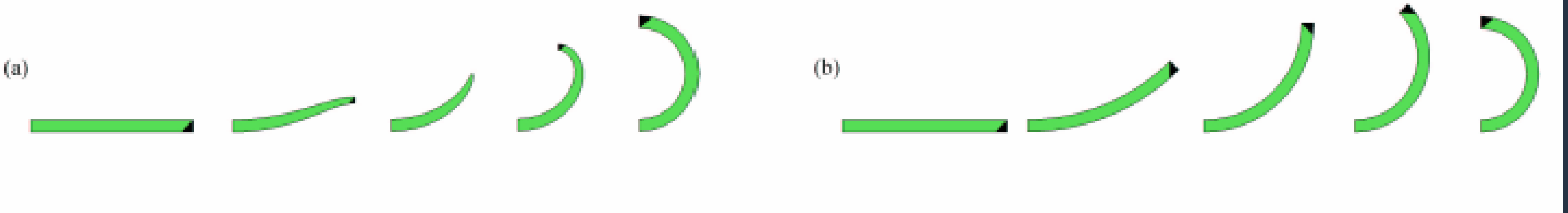

Figure 2: Morphing using (a) linear interpolation, (b) strain field interpolation

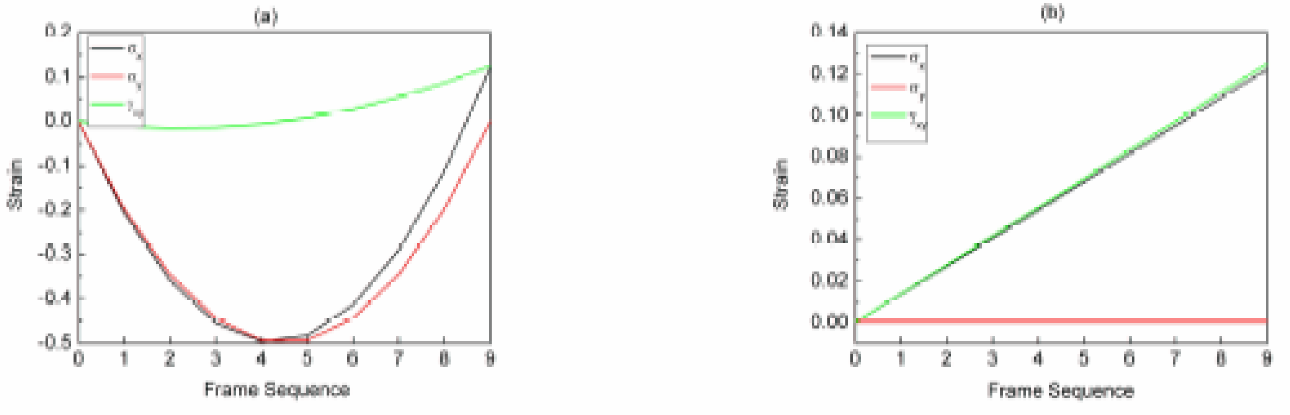

Figure 3: Strain versus time for (a) linear interpolation, (b) strain field interpolation

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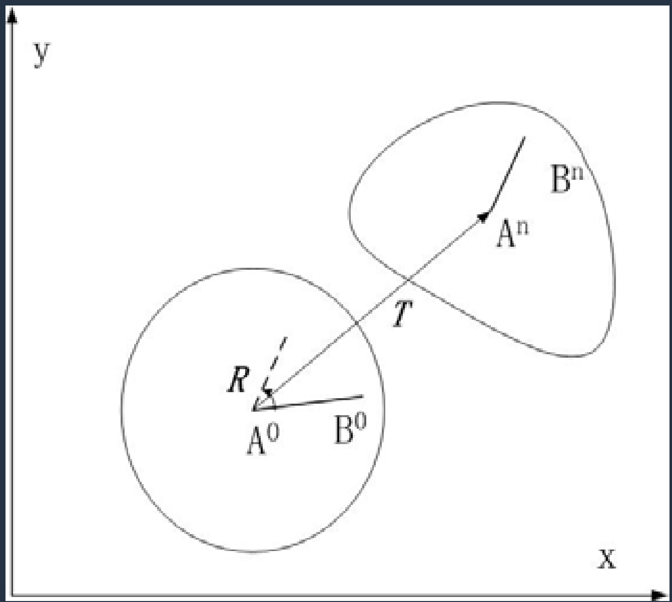
- From Fig2 and Fig3, we can see that “strain” is a very effective tool to analyze object deformation. The key factor for a smooth deformation process is that the curve of strain against time should not only vary slowly, but also be monotonic.
- Then we view the whole procedure of strain based interpolation method for morphing problem.

Fixing Rigid Body Motion



Rigid Body motion should be separated from *deformation* First.

Select two reference points on source and target objects.

A^i: reference point for translation. B^i: reference point for rotation.

- Move the source and target objects so that A^i coincide with origin. Then rotate B^i around A^i until B^i on the positive x axis.
- We define the vector from A^0 to A^n as T, the angle from A^0A^n to B^0B^n as R. $T_A{}^0$ is the vector from origin to A^0, $R_{A}{}^{0}{}_{B}{}^{0}$ is the angle between A^0B^0 and x axis.

Strain Field Interpolation

- The strain field between source and target object can be calculated by Eqn 1.
- A simple way to get strain field for intermediate frames is to use linear interpolation.

$$\varepsilon_x^t = (1-t)\varepsilon_x^n$$

$$\varepsilon_y^t = (1-t)\varepsilon_y^n \qquad \text{Eqn. 2}$$

$$\gamma_{xy}^t = (1-t)\gamma_{xy}^n$$

- Modified strain field interpolation
  - Eqn 2. can got very good result for those cases with large displacements and small strains.
  - When the strain is also very large, though this method still have good shape showing little distortion, the results usually uneven along time axis.

- Eqn. 3 provided an modified scheme. It ensures the length changes in x and y direction, the angular changes between lines initially in the x and y axis, changes linearly with time. It also constraints the length change ratio in the other directions.

$$\varepsilon_x{}^t = \frac{1}{2}\left[\left[t\sqrt{1+2\varepsilon_x{}^n} + (1-t)\right]^2 - 1\right]$$

$$\varepsilon_y{}^t = \frac{1}{2}\left[\left[t\sqrt{1+2\varepsilon_y{}^n} + (1-t)\right]^2 - 1\right]$$

$$\gamma_{xy}{}^t = tg\left(t\,arctg\,\gamma_{xy}{}^n\right)$$

Eqn. 3

- Error Function
  - Usually the interpolated strain field got by Eqn 2. or Eqn 3. is not conform to *Compatibility Equation* in physics. So we create a error function. We attempt to estimate a position field which corresponds to strain fields that are as close as possible to the ideal (interpolated) strain field.

Error function:

$$W = \sum_{i=0}^{n-1} \int \left(\varepsilon'^{t} - \varepsilon^{t} \right)^{T} \left(\varepsilon'^{t} - \varepsilon^{t} \right) d\Omega \qquad \text{Eqn 4.}$$

ε^{t} is the desired (interpolated) strain field of intermediate frames.

ε'^{t} is calculated from position field by Eqn 1.

The position field that minimize error *W* in Eqn 4 is chosen as the position field of that intermediate frame.

Shape Approximation

- *Finite element method* (FEM) is used to give a mathematical description of arbitrary shape. In FEM, the position of a point inside an element can be expressed using a convex combination of the positions of the element's nodes:

$$x = \sum_{k=1}^{l} N_k x_k \qquad y = \sum_{k=1}^{l} N_k y_k \qquad \text{Eqn 5}$$

N_k is the shape function of element and *l* is the number of nodes of the element.

Optimization Solver

- Substituting Eqn 5 into Eqn 1 and Eqn 4, Eqn 4 can be converted into a finite dimensional problem, whose variables are coordinates of all nodes in the desired intermediate frame. The problem of minimizing *W* can be solved by any optimization method.

Equation Solver

- For efficiency, we would like equation solver than optimization solver.
- By differentiate Eqn 4., we can translate the optimization problem to nonlinear equation solving problem:

$$\phi(V) = \sum_{i=1}^{m} \int \left(B^T \varepsilon'^t - B^T \varepsilon^t \right) d\Omega_i = 0 \quad \text{Eqn. 6}$$

B is the strain matrix, which connect element strain and element verteices position vector. *V* is the verteices coordinate vector.

- The derivative (tangent) matrix of Eqn. 6 is given by:

$$K_t = \sum_{i=1}^{m} \int \left[B^T B + G^T (S' - S) G \right] d\Omega_i$$, where

$$S = \begin{bmatrix} \varepsilon_x I_2 & \gamma_{xy} I_2 \\ \gamma_{xy} I_2 & \varepsilon_y I_2 \end{bmatrix} \quad G = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix} \quad G_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} & 0 \\ 0 & \frac{\partial N_i}{\partial x} \end{bmatrix}$$

- Eqn 6. can be solved by Newton-Raphson method:

$$K_l^t\left(V_{l+1}^t - V_l^t\right) = -\Phi\left(V_l^t\right)$$

The coordinate vector of previous frame V_{i-1} is used as initial value for intermediate frame $V_{i.}$

- If the source shape and target shape differ greatly and the number of frames inserted is low, direct use of the Newton-Raphson method can lead to triangle flip.
- There are two ways to solve this problem: using Continuation method or simply Insert more frames.

Interpolate Rigid Body Motion

- We have now intermediate frames which have ignored rigid body motion. We need to add the interpolated rigid body motion back.
- Each frame is rotated by the angle:
  - $R_i = R_A{}^0{}_B{}^0 + t_i * R$
- Then Each frame is translate by the vector:
  - $T_i = T_A{}^0 + t_i * T$

Result

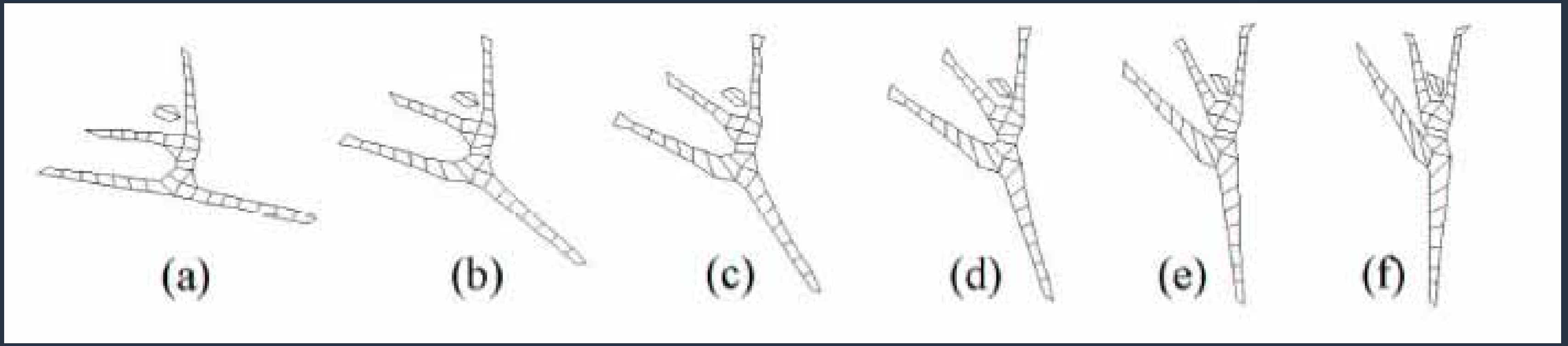


Figure 4. Morphing of a gymnast pattern

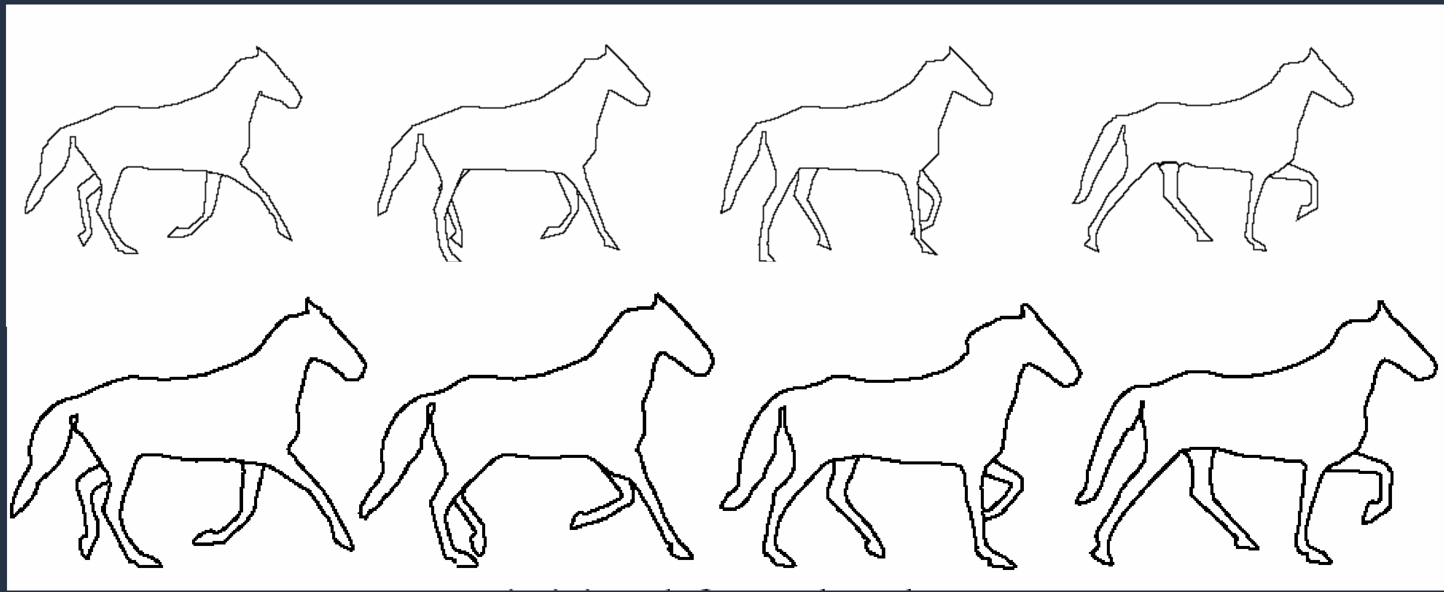


Figure 5. Morphing of a horse

The two left legs were calculated separately.

Demo


MOrphing Based on
Strain Field Interpolation
A to B to C
CASA 2004


MOrphing Based on
Strain Field Interpolation
From Camel to Ox
CASA 2004

Create Isomorphic Meshes

- This part of work is to create isomorphic meshes for source and target objects, which were used in the previous morphing process calculating.
- The aim of our work is to create isomorphic meshes with less triangles than some other method did. We will use camel-ox in our demo as an example.

- Choosing points and patching
  - From two objects with texture, border points and inner points were first selected.

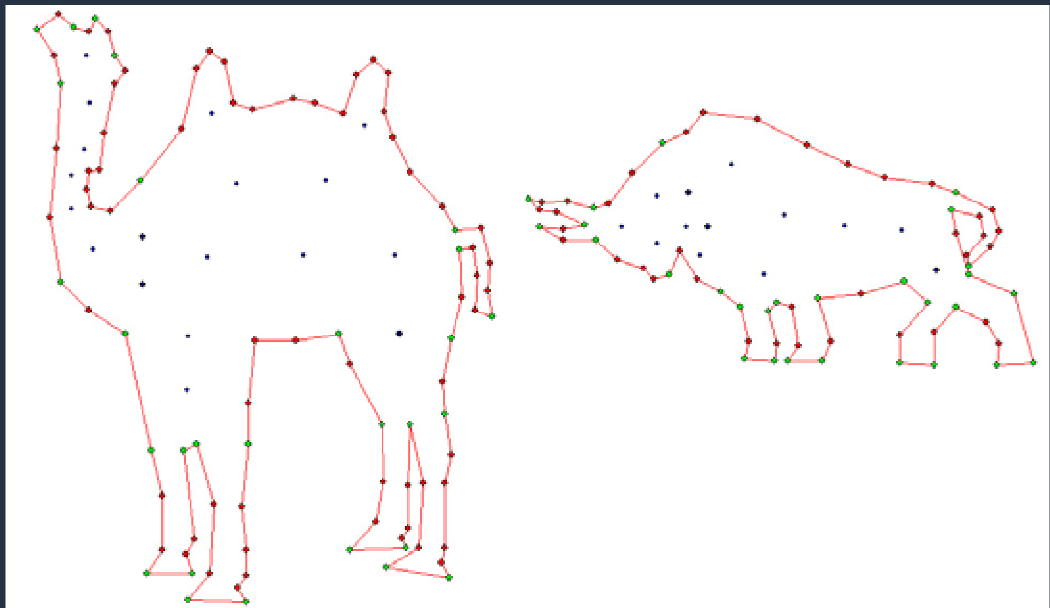


Figure 6

- Then some border and inner points were selected as corresponding (anchor) points.

- All inner anchor points and some border anchor points were selected as feature points.
- Triangulate source and target objects.

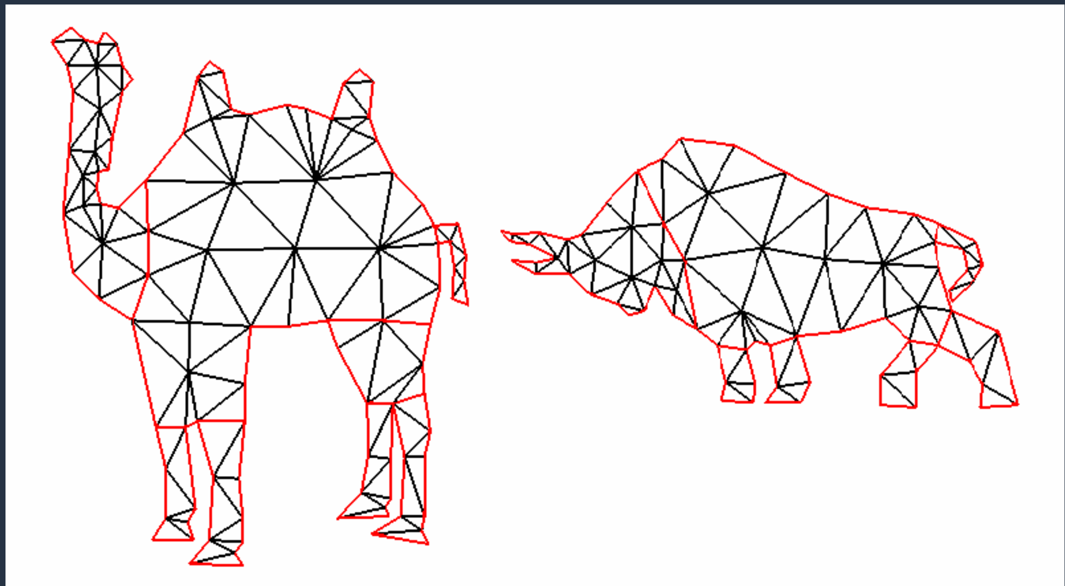


Figure 7

- With feature points, source and target objects were to be dissected into several topological equivalent patches (as in Fig 7) . Praun’s method (siggraph 2001) can be used to create such equivalent patches, or by interactive way.
- The purpose of such dissection is to split objects with large concavities to several less concave objects. Besides, it can split objects whose genius is none zero to several patches of genus zero.

- Parameterization
  - Sederberg's methods were used to create consistent borders for each pair of corresponding patches.
  - If needed, added some inner points near the concave angle of the patch. Then re-triangulate each patch.
  - Using mean value method to parameterize each pair of patches onto n-gon surrounded by a circle.

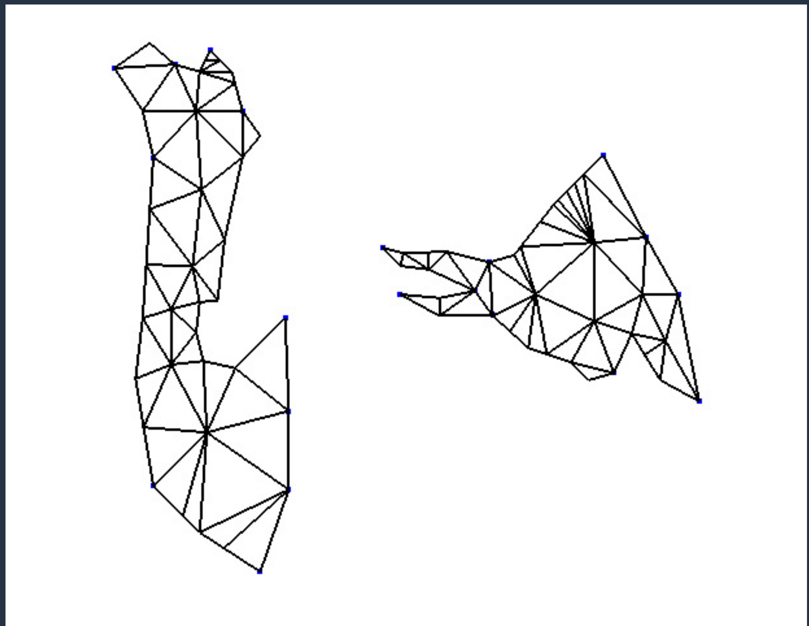


Figure 8. Head patches

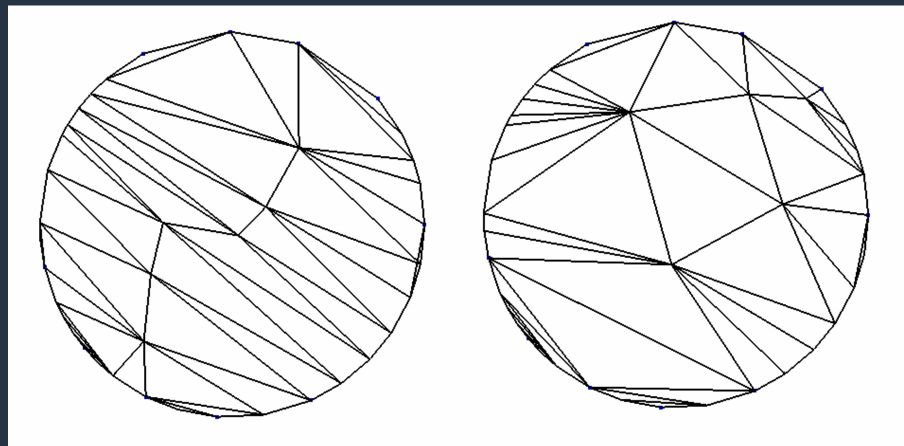


Figure 9. Parameterized on n-gon

- Merge
  - Merge the points from the source and target patch parameterized on n-gon, and ignore the edges. Then triangulate the merged points on n-gon.

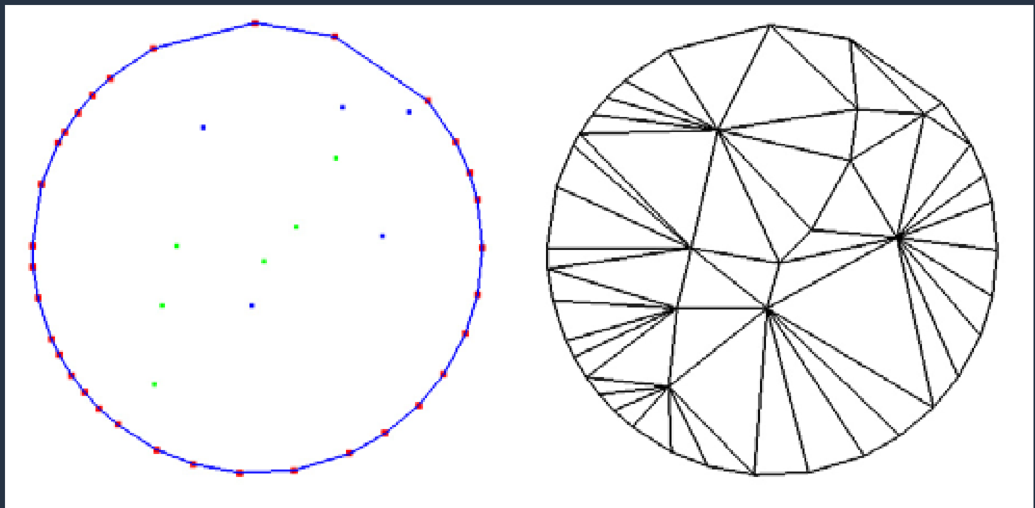
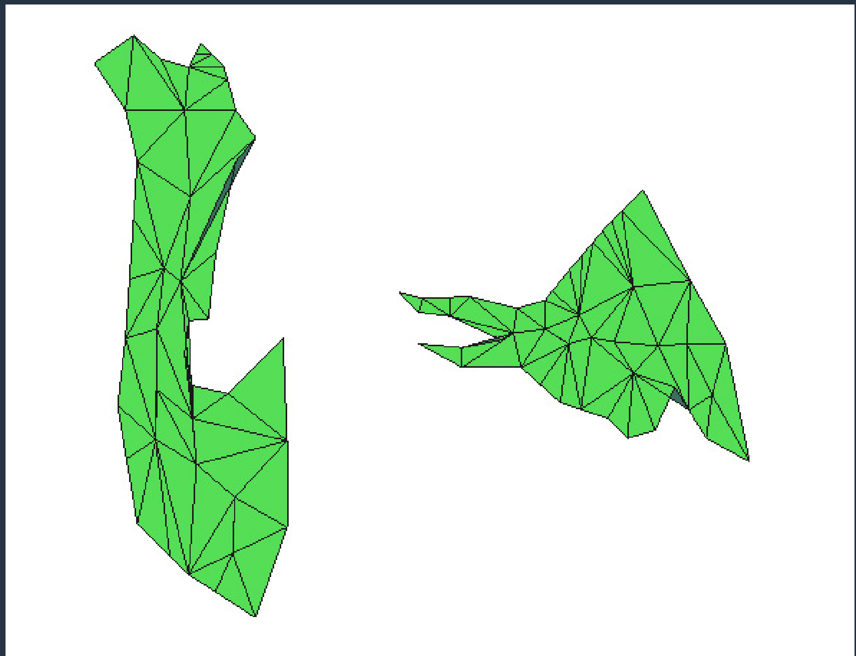


Figure 10. Merged points and re-triangulate results.

- Map back and flip operation
  - Map the merged triangulation on n-gon back onto the source and target object.
    - If there are flipped triangles, try flip operation to see if it can reverse the orientation and ensure the orientation on corresponding triangles are positive. Usually it works.
    - If it does not work, because there is inadequate inner points were added near any concavity of the patch. Added more points near in the patch and repeat patch parameterization.



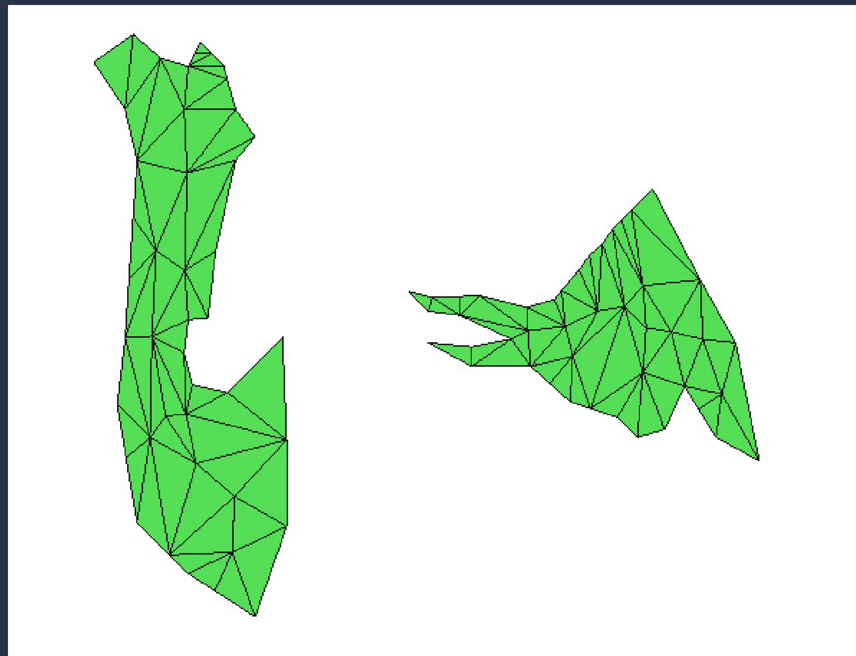


Figure 11. Mapped the head patches back, there are 3 triangles has negative orientation.

Figure 12. After flip operation and optimization

- Merge patches and optimization
  - Merge the patches together to give isomorphic meshes for source and target objects.
  - Move the position of vertices and make flip operation to improve the triangulation quality. This step can also be made after mapping back operation for each pair of patches.

- The final results show that our method can create isomorphic meshes with less triangles than method of Alexa.

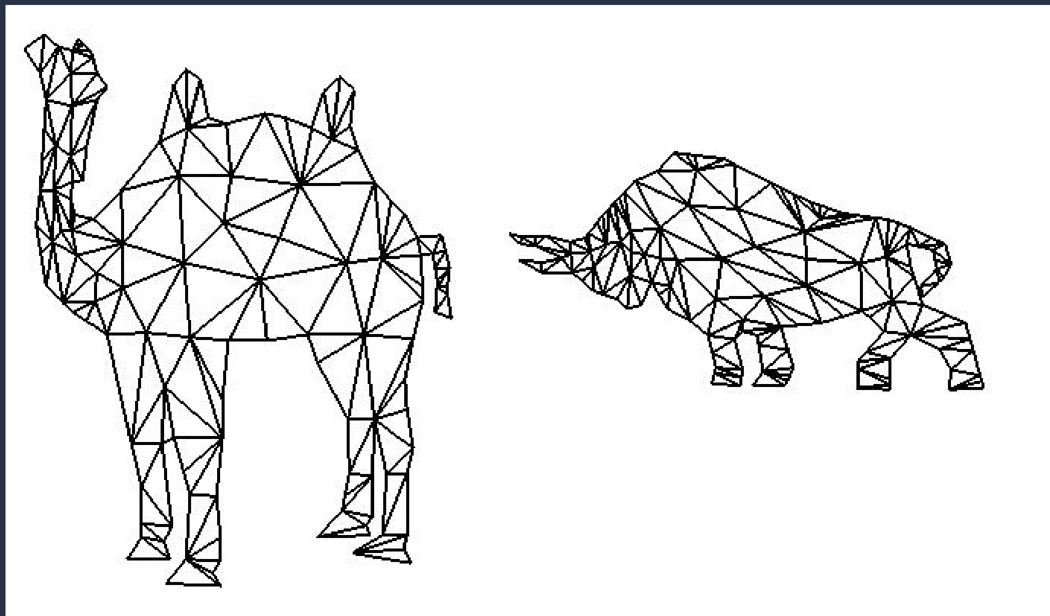


Figure 13. Isomorphic meshes of camel-ox.

- Deficiency of this method
  - Sometimes some manual work is needed to add more inner points in patches.

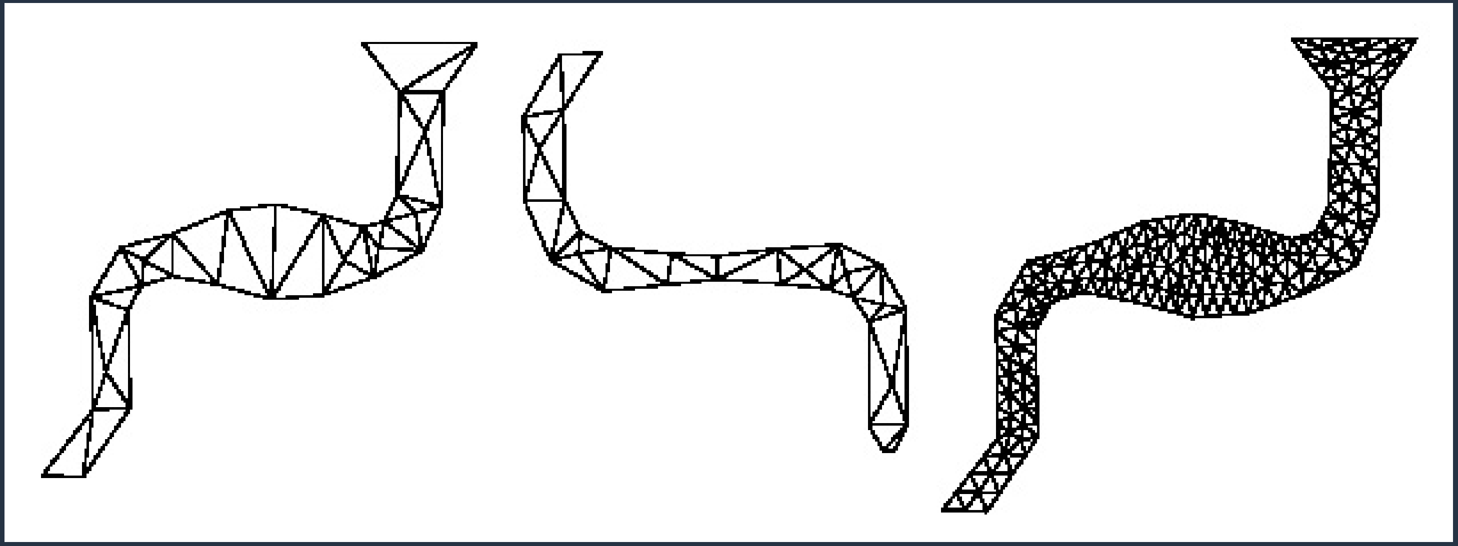


Figure 14. Original mesh and isomorphic mesh given by Alexa

Conclusion

- There are two contributions were given by this paper.
  - A morphing method based on strain field interpolation was proposed. Strain gives deformation a quantity measurement.
  - A method to create isomorphic meshes with less triangles was proposed.

Thanks ！

Appendix

- In the paper of Alexa, object was decomposed into triangular meshes and affine transform matrix was decomposed into rotation and stretch matrix, whose intrinsic purpose is to use local deformation instead of global displacement.
- Though the paper try to connect matrix decomposition with “rigid” in physics, it is far from the real physics concept and it is very hard to be used to analyze deformation in quantity.