

Computer Graphics

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BRDF(双向反射分布函数)



- BRDF
 - Bidirectional Reflectance Distribution Function
 - Describe how light is reflected from a surface
- Preliminary for BRDF
- BRDF: definition and Properties
- **BRDF Models**
- BRDF Measurement





- Illumination can be classified as local or global.
 - Local illumination is concerned with how objects are directly illuminated by light sources.
 - Global illumination includes how objects are illuminated by light from locations other than light sources, Including by reflection of other objects and refraction through objects.





- Illumination can be classified as local or global.
 - Local illumination is concerned with how objects are directly illuminated by light sources.

Today's Topic: a physical description of how light is reflected from a surface, which is known as BRDF.

Preliminary



- Before introducing BRDF, we review some preliminary concepts.
 - Spherical Coordinate (球面坐标)
 - Solid Angle (立体角)
 - Foreshortened Area (投影面积)
 - Radiant Energy (光能)
 - Radiant Flux (光通量)
 - Irradiance (辉度)
 - Intensity (发光强度)
 - Radiance (光亮度)



 Since light are mostly expressed in terms of directions, it is generally more convenient to describe them by spherical coordinates rather than by cartesian coordinate vectors. Z[†]





Spherical Coordinate (球面坐标)

- Since light are mostly expressed in terms of directions, it is generally more convenient to describe them by spherical coordinates rather than by cartesian coordinate vectors.
- As illustrated in the figure, a vector in spherical coordinates is specified by three elements.
 - magnitude r denotes the length of the vector.
 - Θ measures the angle between the vector and the z-axis,
 - ψ represents the counterclockwise angle on the x-y plane from the x-axis to the projection of the vector onto the xy plane.



• Relationship between Cartesian(笛卡尔) and spherical coordinates

 $-(\mathbf{x},\mathbf{y},\mathbf{z}) \longleftrightarrow (\mathbf{r},\boldsymbol{\varTheta},\boldsymbol{\psi})$

- Conversion
 - r = sqrt($x^2+y^2+z^2$)
 - $\Theta = \operatorname{acos}(z/r);$
 - $\psi = \operatorname{atan2}(y,x);$
 - $\mathbf{z} = \mathbf{r} \cos(\boldsymbol{\Theta});$
 - $y = r \sin(\Theta) \sin(\psi);$
 - $\mathbf{x} = \mathbf{r} \sin(\boldsymbol{\Theta}) \cos(\boldsymbol{\psi});$



Solid Angle(立体角)



- Light generally arrives at or leaves a surface point from a range of directions that is denoted by solid angles. solid angles represents a 3D generalization of angle formed by a region on a sphere.
- Max value of a solid angle is 4π , which is given by a sphere.

$$d\omega = \frac{ds}{r^2}$$





Solid Angle(立体角)



For a differential solid angle described by differential angles dθ,dφ in the θ,φ directions, its differential area dA on the sphere is

$$dA = (rd\theta)(r\sin\theta d\varphi) = r^2\sin\theta d\theta d\varphi$$

• From the solid angle definition, the differential solid angle is given by:

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$



Foreshortened Area (投影面积)

- The apparent area of a surface patch according to the angle at which it is viewed
- For a surface patch of area A, its foreshortened area from direction θis given as A cos(θ), since its apparent length in the x direction is scaled by cos(θ).

$$Area = A\cos\theta$$



Radiant Energy (光能)



- An amount of light energy can be thought of as the total energy of a collections of photons (光子), the quantity is referred to as radiant energy.
- Radiant energy is denoted by Q and measured in Joule (J)



Radiant energy Q

Radiant Flux (光通量)



- Radiant energy does not stay fixed in one position, but moves
- Radiant Flux describes:
 - The flow of energy through an area per unit time
 - To represent radiant energy in motion, we have

$$\Phi = \frac{dQ}{dt}$$
– Measure in watt (W)(瓦特)

 1 watt (W) is equivalent to one Joule(焦耳) per second



Radiant flux Φ





- To describe interactions of radiant energy and a surface
 - We represent the incoming radiant flux per unit surface area(单位面积的光通量) as

$$E = \frac{d\Phi}{dA}$$
- measured in W/m



Irradiance E

Intensity (发光强度)



- Radiant Flux (光通量) through a point can't be described by irradiance, since a point consists of Zero area.
- So we use Intensity, described as:
 - Radiant flux with respected to solid angle

$$I = \frac{d\Phi}{d\omega}$$



Intensity I





Radiance L

- Radiant flux is always described in terms of both surface area and solid angle
- The measure of flux entering or leaving a surface per solid angle per unit foreshortened surface area is called
 Radiance which can be measured by

 $W/(sr \cdot m^2)$

Radiance (光亮度)



 Note that it is the foreshortening of surface area in radiance, which is in contrast to the other area-based quantity irradiance (辉度) that are non-directional.



Foreshortening

• An intuitive way to explain this foreshortening factor in radiance is to consider radiance as a flow of energy from a source to a receiver as exhibited in the figure for a receiver perpendicular to the flow direction.





• Defined respect to the apparent surface as:

$$L = \frac{d\phi}{dA\cos\theta \, d\omega}$$

- Foreshortening of Area
 - the area perpendicular to the flow direction



Foreshortening

Irradiance (辉度) from radiance (光亮度)

• Irradiance is the integral of radiance over hemisphere

$$\frac{d\Phi}{dA} = E = \int_{\Omega} L(\omega) \cos\theta d\omega$$

- $-\Omega$ is the hemisphere of incoming directions
- -L is the incoming radiance



- BRDF Definition
- BRDF Properties
 - -reciprocity (可逆性)
 - -energy conservation (能量守恒)
 - rendering equation (绘制方程)

BRDF Definition



- Appearance of a surface results from its reflection of light from the surrounding environment towards the viewer.
- In most computer graphics algorithms, reflections are modeled by Bidirectional Reflectance Distribution Function (BRDF)



- BRDF is:
 - A 4D function that expresses the ratio of reflected radiance (光亮度) towards a viewing direction to the irradiance (辉度) from an incoming direction: $f(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_r)}{dE_i}$
 - Expressed in terms of incoming radiance:

$$f(\omega_i \to \omega_r) = \frac{dL_r(\omega_r)}{L_i(\omega_i)\cos\omega_i d\omega_i}$$

BRDF Definition



- Measured in units of BRDF can be (sr)⁻¹
- Reflectance Geometry:





- Reciprocity
 - Reciprocity is arise from the Helmholtz reciprocity rule(光路可逆性)
 - Exchanging the directions of incoming and outgoing does not result in a change in BRDF $f(\omega_i \rightarrow \omega_r) = f(\omega_r \rightarrow \omega_i)$

BRDF Properties: energy conservation(能 量守恒)

- Another physical law that governs BRDFs is conservation of energy, which requires the energy of scattered light to be equal to the energy of incoming light.
- Conservation of energy:

$$Q_{incoming} = Q_{reflected} + Q_{absorb} + Q_{transmitted}$$

• We have:

$$Q_{reflected} \leq Q_{incoming}$$



 So the BRDF must satisfy the following condition according to energy conservation:

$$\int_{\Omega} f(\omega_i \to \omega_r) \cos \omega_r d\omega_r \le 1$$

BRDF Properties: rendering equation

- BRDF is formulated in a manner that accounts for the fact that the total radiance observed in direction_r arises from incoming radiance from multiple directions.
- The viewed radiance is computed as an integration of light reflected from different incoming angles:

$$L_r = \int_{\Omega} f(\omega_i \to \omega_r) L(\omega_i) \cos \omega_i d\omega_i$$

• The above is referred to as the rendering equation





- For convenient and efficient use of 4D BRDFs, they are customarily represented by parametric BRDF models.
- The numerous models of BRDF that have been proposed can be classified into three general categories
 - Empirical Models (经验模型)
 - Physical-based Models (基于物理的模型)
 - Data-driven Models (数据表达的模型)



- Empirical Models (经验模型)
 - designed experimentally to yield a reasonable appearance at a low computation cost.

• Physical-based Models (基于物理的模型)

- describe reflectance according to the geometric and optical properties of a material
- Data-driven Models (数据表达的模型)
 - describe reflectance directly based on tabulated data

Empirical Models (经验模型)



- Empirical Models
 - Given in Simple forms that allow for rapid computation of reflectance
 - Ignore material properties, only provide a rough approximation of actual reflectance,
 - do not satisfy physical principles such as
 Helmholtz reciprocity and energy conservation
 - nevertheless, they are the most commonly used ones because of practical advantages.



- Lambertain's Model: the most basic model
 - incoming light is reflected equally in all directions
 - BRDF function f is a constant among all directions
- Albedo(反射率) ho
 - The ratio of exit irradiance to incoming irradiance

$$L_{r}(\omega_{r}) = \int_{\Omega} fL_{i}(\omega_{i}) \cos \omega_{i} d\omega_{i}$$

= $f \int_{\Omega} L_{i}(\omega_{i}) \cos \omega_{i} d\omega_{i} = fE$
 $\rho = \frac{M}{E} = \frac{\int_{\Omega} L_{r}(\omega_{r}) \cos \omega_{r} d\omega_{r}}{E} = \frac{fE\pi}{E} = \pi f$ Lambertian

Empirical models: Lambertain



- Lambertain's Model
 - Well representing the reflectance of objects that exhibit purely diffuse reflection (漫反射)
 - However, for those materials, such as metal(金属),
 with a significant specular component, lambertain's
 model could not represent it well
 - Because of its simplicity and reasonable modeling, it has been incorporated into several more sophisticated models, such as Phong Model



Lambertian

Empirical models(2): Phong



- Phong Model
 - is to empirically adds a specular lobe in the mirror direction of the incoming direction
 - Expressed as:

$$f(l \rightarrow v) = \rho_d + \rho_s \frac{(r \cdot v)^s}{(n \cdot l)}$$

– Reciprocity property does not hold

 $f(l \to v) \neq f(v \to l)$



Phong



Phong Model

Phong

- is to empirically adds a specular lobe in the mirror direction of the incoming direction
- Expressed as:

$$f(l \to v) = \rho_d + \rho_s \frac{(r \cdot v)^s}{(n \cdot l)}$$

where ρ_d ; ρ_s denote albedos of diffuse and specular reflections, respectively, and *s* signifies a shininess factor that modulates the sharpness of specular reflections.

Empirical models: Phong



- Phong Model
 - Although the Phong Model lacks a physical explanation and has been shown to be inaccurate for a broad range of materials, it has long been the most popular BRDF model in computer graphics
 - Largely due to its efficiency in computation and acceptable

appearance



Phong
Extensions of Phong Model



- Most extensions are proposed towards faster computation
- Blinn-Phong Model
 - utilize the bisector(角分线) h of light direction l and view direction v instead of the mirror direction r of l
 - simpler computation

$$f(l \to v) = \rho_d + \rho_s \frac{(n \cdot h)^s}{(n \cdot l)}$$

where h = (v+l)/2

Extensions of Phong Model



- Modified Phong Model
 - Phong model doesn't hold reciprocity property

$$f(l \to v) = f(v \to l)$$

- We may cancel out the equation's cosine term

$$f(l \to v) = \rho_d + \rho_s (r \cdot v)^s$$

to have reciprocity property



Phong

Extensions of Phong Model



• *Fast Phong Shading* : Tabulate and Interpolate the specular exponent function (Bishop and Weimer, SIGGRAPH 86)

$$(r \cdot v)^s$$

- It is known that exponential computation is costly
- to reduce the computation of the specular component
- We can tabulate the exponential function and interpolate it



Phong

Results of Phong Model





Phong model examples_1





Phong model examples _2





Physical-Based Models (物理模型)

- Physical-Based Models
 - While empirical models originate from the intuition and practical experience of the designers, physical-based models are built on scientific knowledge of light interaction.
 - These models Incorporate various geometric and optical properties of materials with the goal of representing real-world materials as accurate as possible



- Physical-Based Models
 - Generally established from fine-level geometric structure called surface roughness (表面光泽度)
 - Roughness:
 - from the microscopic view, almost no surface is perfectly smooth
 - Micro-scale surface geometry is modeled by a collection of planar microfacets
 - Roughness is represented as a statistical distribution of microfacet orientations





- Fresnel Term
 - In reality, we found that specularity increased near grazing angles



 The amount of incident light that reflects is determined by the Fresnel reflection formulas from Maxwell's equations for electromagnetic waves.



Physical-Based Models (物理模型)

• Fresnel Term

Fresnel Reflectance Geometry





Physical-Based Models (物理模型)

- Fresnel Term
 - defined as below:

$$F_s = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta}$$
$$F_p = F_s \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}$$

$$2a^{2} = \sqrt{(\eta^{2} - \kappa^{2} - \sin^{2}\theta)^{2} + 4\eta^{2}\kappa^{2}} + (\eta^{2} - \kappa^{2} - \sin^{2}\theta)$$
$$2b^{2} = \sqrt{(\eta^{2} - \kappa^{2} - \sin^{2}\theta)^{2} + 4\eta^{2}\kappa^{2}} - (\eta^{2} - \kappa^{2} - \sin^{2}\theta)$$

$$\eta = \frac{\eta_m \eta_a + \kappa_m \kappa_a}{\eta_a^2 + \kappa_a^2}$$
$$\kappa = \frac{\eta_a \kappa_m - \eta_m \kappa_a}{\eta_a^2 + \kappa_a^2}$$

- Fresnel Reflectance: $F = (F_p + F_s)/2$



- Cook-Torrance Model
 - Proposed by Cook and Torrance in [A Reflectance Model for Computer Graphics, ACM SIGGRAPH, 1981]
 - The earliest physical-based BRDF model used in computer graphics
 - An adaptation of the Torrance-Sparrow model developed by applied physicists [*Theory for O*®-*Specular Reflection from Roughened Surfaces*, J.
 Optical Society of America, 1975]



- Cook-Torrance Model
 - microfacets are assumed to be mirror reflectors
 - microfacets are assumed to be arranged as Vshaped grooves as exhibited in the figures





Directions



- Cook-Torrance Model
 - combining a Lambertian diffuse term with a specular term based on microfacet reflections:

$$f_{\lambda} = d \frac{\rho_d}{\pi} + s F_{\lambda} \frac{DG}{4\pi (n \cdot l)(n \cdot v)}$$

- where F_{λ} is the Fresnel factor, D is the distribution function of the microfacet normal, G is the geometrical attenuation factor, and s,d are specular and diffuse coefficients



- Distribution Function of the microfacet normal
 D
 - since microfacets are mirror reflectors, only those with a normal along the bisector direction h contributes to specular reflection



depends on roughness of surface

– the Bechmann distribution is used:

$$D(h) = \frac{1}{m^2 \cos^4 \beta} e^{-\frac{\tan^2 \beta}{m^2}}$$

where m is the roughness value of the surface, β is the angle between n and h



- Geometric Attenuation Factor G
 - Accounts for masking of microfacets from viewing direction and for shadowing from the light direction
 - To exclude masked and shadowed microfacets, G is defined as:

$$G(n,l,v) = \min\left\{1, \frac{2(n \cdot h)(n \cdot v)}{(v \cdot h)}, \frac{2(n \cdot h)(n \cdot l)}{(v \cdot h)}\right\}$$





- Geometric Attenuation Factor G
 - This consideration of microfacet masking and shadowing allows certain reflectance features to be generated
 - One is off-specular reflection
 - Another is retroreflection





- Off-specular reflection
 - Specular lobe is not centered on the mirror direction
 - This characteristic is typical of rough surfaces





- Retroreflection
 - A significant amount of light is directed back towards the light direction.
 - See the full moon, the edges appear as bright as the center





Results of Cook-Torrance Model (Compared to Phong)



Plastic-looking copper rendered using Phong model



A Copper Vase with a more metallic appearance



More Cook-Torrance Results





Other Physical-Based Models (物理模型)

- BRDFs can be classified into two groups
 - Isotropic (各向同性)
 - Reflectance independent of rotation about a given surface normal
 - Random surface microstructure
 - Anisotropic(各向异性)
 - Reflectance changes with rotation around a given surface normal
 - Patterned surface microstructure
 - Brushed metal, satin, hair



- However, None of Phong or Cook-Torrance BRDF models could handle anisotropic effects
- Now we introduce Another BRDF Model: Ward Model



Ward Model



- Ward Model:
 - Proposed by Ward in 1992[Measuring and Modeling Anisotropic Reflection, ACM SIGGRAPH, 1992]
 - presents a more general surface normal representation as a collection of ellipsoids(椭圆体), that allows for anisotropic reflection
 - However, exclude the Fresnel factor and geometric attenuation factor, make it viewed as somewhat empirical

Ward Model



(4)

• Isotropic Ward Model, defined as:

$$\rho_{bd,iso}(\theta_i,\phi_i;\theta_r,\phi_r) = \frac{\rho_d}{\pi} + \rho_r \cdot \frac{1}{\sqrt{\cos\theta_i \cos\theta_r}} \cdot \frac{\exp[-\tan^2\delta/\alpha^2]}{4\pi\alpha^2}$$

where:

- ρ_d is the diffuse reflectance
- ρ, is the specular reflectance
- δ is the angle between vectors \hat{n} and \hat{h} shown in Figure 5 α is the standard deviation (RMS) of the surface slope
- Replaced (Fresnel factor and geometric attenuation factor) with a single normalization factor that simply ensures the distribution will integrate easily over the hemisphere

Ward Model



• Anisotropic Ward Model, Defined as:

$$\rho_{bd}(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi} + \frac{1}{\sqrt{\cos\theta_i \cos\theta_r}} \cdot \frac{\exp[-\tan^2\delta(\cos^2\phi/\alpha_x^2 + \sin^2\phi/\alpha_y^2)]}{4\pi\alpha_x \alpha_y}$$
(5a)

where:

 ρ_d is the diffuse reflectance

 ρ_s is the specular reflectance

 α_x is the standard deviation of the surface slope in the \hat{x} direction

 α_{y} is the standard deviation of the surface slope in the \hat{y} direction

 δ is the angle between the half vector, \hat{h} and the surface normal, \hat{n} .

 ϕ is the azimuth angle of the half vector projected into the surface plane.

Results of Ward Model



Photograph Isotropic Ward Anisotropic Ward



More BRDF Models



- More BRDF Models
 - Oren-Nayar model [Generalization of Lambert's Reflectance Model ACM SIGGRAPH, 1994]
 - Treats microfacets as Lambertian reflectors rather than as mirrors
 - Poulin-Fournier model [A Model for Anisotropic Reflection, ACM SIGGRAPH, 1990]
 - represents microfacet normals using groups of parallel cylinders, also handle anisotropic effects

More BRDF Models



- Wave optic related Models (波动光学)
 - Developed from principles of wave optics
 - Materials with a microfacet size comparable to wavelengths of light
 - Based on diffraction(衍射) theory, two main work [He, et al., A Comprehensive Physical Model for Light Reflection, ACM SIGGRAPH, 1991] and [Stam, Diffraction Shaders, ACM SIGGRAPH, 1999]
 - Although have great power, but their high complexity limits their use

More BRDF Models



• Effects of the model wave optic related (波动光学)

a picture of a CD illuminated by a directional light source. Notice that all the highlights appear automatically in the correct places



Data-Driven BRDF Models



- Measured BRDF of a large set of materials, and record them as high-dimensional vectors
- From this data, a lower dimensional manifold is computed using dimensional reduction methods
- **Representative work:** Matusik et al. [*A Data-Driven Reflectance Model*, ACM SIGGRAPH, 2003]

Comparison of BRDF Reprentations



- Empirical Models:
 - Computationally simple, visually acceptable
- Physical-Based Models:
 - Parameters physically meaningful
 - Built on scientific knowledge
- Data-driven Models:
 - Flexibility, no assumption about materials
 - Large data set, usually needs data reduction methods to compress data
 - Needs interpolating data, missing high light parts



- Some demo
 - Real-time BRDF editing in complex lighting
 - Time-Varying BRDFs

BRDF Measurement



- Motivation
 - For modeling of materials with unknown reflectance properties and for obtaining highly realistic appearance
 - The process of recovering BRDF and other scene attributes, which is sometimes referred to as inverse rendering
 - We give an overview of BRDF measurement
 - Measurement devices
 - Practical issues

BRDF Measurement



- Measurement Devices
 - Since the BRDF is a function of lighting and viewing directions, measurement involves sampling 2D space of light directions and 2D space of viewing directions



BRDF Measurement



- Measurement Devices
 - Fix material sample, move light source and camera
 - Fix light source, move the camera and tilt the sample
 - Fix light source and sample, move the camera, if the target object consists of a single material and has a known convex geometry




- Light Stage
 - Debevec et al. [SIGGRAPH 2000]
 - BRDFs are measured from a fixed object or person
 - Images are captured from a few viewing directions
 with about 2000 different lighting directions
 - Distinct BRDFs can be measured for all the image points simultaneously.





Measurement Device: Light Stage

• Re-illuminate the face





• Gonioreflectometer





Measurement Device: Gonioreflectometer

- Gonioreflectometer
 - Utilize a half-silvered hemisphere to view a material sample from multiple directions at the same time
 - A light source from outside the hemisphere illuminate the sample
 - A camera with a fisheye lens captures the entire hemisphere at once
 - Eliminating two degrees of freedom, only light source needs to move

Practical Issues:



- Camera Calibration (相机参数矫正)
 - To measure a BRDF, the camera must be calibrated in a few respects
 - The external parameters, such as its location and orientation; the internal parameters, includes its focal length and camera center
 - Using Zhang's method [Zhang00, a flexible new technique for camera calibration, IEEE PAMI] by capturing several pattern images

Practical Issues:



• Camera Calibration (相机参数矫正)





• Next week's topic:

Ray Tracing

More on course project



Registration

• Acquisition and modeling pipeline



Bunny Entity



Point Patches



Bunny Mesh Model

Mesh Generation Remeshing Mesh Repair Topology Editing



Point Representation



• 激光扫描设备: ViViD910







Thanks!